№ 3 (130) / 2025

Vol. 130, № 3 / 2025

Техніка, енергетика, транспорт АПК

UDC 631.319.4
DOI: 10.37128/2520-6168-2025-3-11

MATHEMATICAL MODEL OF CONTACT INTERACTION BETWEEN A ROTATING WORKING BODY AND PLANT STEMS FOR THEIR SHREDDING

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The paper proposes a mathematical model of contact interaction between the blade as the working body of a rotary shredder of plant residues and plant stems removed from the field surface. The method of removing such vegetation involves shredding it and scattering it over the field surface. The practical purpose of implementing the mathematical model is to determine the minimum circular speed of the working body when it interacts with the stems.

The mathematical model is based on the assumption that a plant stem can be modelled using Euler-Bernoulli beam theory. To theoretically reproduce the impact pulse interaction of the blade on the stem, a load approximation was used with delta functions having a narrow peak section. The model allows the deformation and oscillatory processes that occur in the stem after impact by the blade of the rotating working body to be evaluated. A solution to the Euler-Bernoulli equation for impulse impact has been constructed. The obtained expressions for the transverse displacements of the beam axis point have a modal decomposition for impulse impact. Since real stems have actual energy dissipation, modal damping, expressed by the corresponding coefficient, has been introduced into the model. For the numerical solution of the problem, a truncated series is considered: 10 modes for smooth pulses and 50 modes for very short pulses. This number of modes already gives a good approximation of the displacement and depends on the geometry and parameters of the interacting bodies.

The result of the numerical experiment is to obtain the values of the maximum bending moments that cause bending stress and compare them with the critical stresses that cause stem failure. For the reliability of the results obtained, the safety factor for bending has been increased to 1.2, which will ensure guaranteed stem failure. Based on this, the minimum speed of interaction between the blade and the stem when the breaking condition is met was found. This speed is 8-10 m/s, at which the stems will only be broken, but not crushed. Imposing the condition of shredding the stems into particles with a length of approximately 0.03 m, a series of peripheral circumferential speeds of the rotary working element's blade was obtained for various ground speeds of the machine. For example, at a forward speed of the unit of 3.0 km/h (0.83 m/s), the circular speed of the blade should be 12-14 m/s; for a forward speed of the unit of 4.0 km/h (1.11 m/s) – 16-18 m/s; for 5.0 km/h (1.39 m/s) – 20-22 m/s; for 6.0 km/h (1.67 m/s) – 24-28 m/s. Thus, the implementation of the mathematical model of the interaction between the blade of the rotary working element and the stems of the plants being removed, based on the Euler–Bernoulli beam model, allowed us to substantiate the minimum circumferential speed of the working element for various forward speeds of the unit and the required length of the shredded material.

Key words: *stem, shredding, mulcher, harvesting, bending, stress, moment, speed, rotary working element, mathematical model, impact, energy, experiment, process, destruction, shearing.*

Eq. 25. Fig. 3. Ref. 17.

1. Problem formulation

Many crop harvesting technologies involve the removal of crop tops or stalks. Typical working bodies that perform this technological operation are rotary blade-type machines. In addition, they are also used to clear the field surface of weeds, shred the stems of tall mowed plants, etc. Plant residue mulchers or other simple or combined rotary machines are built on this principle. Practical experience shows that for high-quality performance of technological operations, such working bodies must have high rotation frequencies of working rotors, since they operate on the principle of non-supporting cutting of plants. On the other hand, the high

Article received 24.10.2025. Article accepted 08.12.2025. Article published 25.12.2025.

angular velocity of a massive rotor means significant drive power consumption, significant loads on structural elements, risks of resonance modes due to imbalances, etc. Therefore, the assessment of the contact interaction of the working body of the blade with the object of destruction, based on the determination of the minimum circular speed of the blade, is a relevant problem that is being developed in this study.

2. Analysis of recent research and publications

Many scientific works are dedicated to the issues of plant residue shredding and their incorporation into the soil [4, 5, 8, 12, 14, 15, 17]. However, the calculation basis here relies on the principle of supported cutting or breakage of stems, mainly of coarse-stemmed plants. As for the theory of plant mass shredding based on the principle of unsupported (or free) cutting, various models are applied in practice [2, 3, 6, 7, 9] that utilize specific plant properties. This makes it possible to ensure an efficient shredding process when designing working elements and assigning them appropriate kinematic operating modes [1, 10, 11, 13, 16]. However, analysis of the works indicates that studies related to the substantiation of parameters for flail rotary working elements that ensure the specified conditions for grinding plant residues at a minimally sufficient rotor rotation frequency have not been sufficiently researched or have an overly complex mathematical apparatus, which hinders their practical application. Therefore, there is a need to develop the theoretical basis for studying the interaction of blade rotary working bodies with plant mass in order to obtain practical calculations that will allow the assignment of energy-saving kinematic modes of operation for such machines.

3. The purpose of the article

The aim of the work is to construct a mathematical model of the impact interaction between the mulcher blade and the plant stem, which allows predicting the mode of its destruction and justifying the energy-saving kinematic mode of operation of such a machine.

4. Results and discussion

To build a mathematical model of the impact interaction between the mulcher blade and the plant stem for predicting the destruction mode, the kinematic and design parameters of the blade and the conditional method of fixing the stem of the plant being removed must be taken into account.

To facilitate understanding of the process, we will initially analyze this interaction based on the isolated impact of the mulcher blade on a singular stem. Then, based on this, it will be possible to complicate the model and consider the interaction with a group of stems and a series of impacts with the beater.

To construct a mathematical model of the described interaction, we must first consider the most likely mechanisms of stem destruction upon impact with the blade.

The first option is local shearing, in which the force of interaction has a tangential component (cutting with slippage), and contact stress occurs at the point of impact with the blade, leading to the cutting of plant tissue. Here, the criterion is to ensure a contact pressure value at which stresses exceeding the shear strength condition arise.

The second mechanism of stem destruction can be brittle or ductile-plastic fracture during bending. To achieve stem destruction by this mechanism, the impact of the blade must create a bending moment at which stresses arise inside the stem that cause the fibres to break.

If the plant has a weak root system that anchors it in the soil, the action of the blade may pull the plant out of the soil by its roots, and this option will be classified as the third mechanism of destruction.

The next mechanism of destruction may be the so-called cutting-chopping. In this case, the working edge of the blade normally contacts the surface of the stem, presses into it, and makes a concentrator. As a result, the stem loses its shape and is destroyed by compression rather than cutting. This option is typical for blunt edges or massive blades and a short-term strong impulse.

It is obvious that the above mechanisms of stem destruction are almost never implemented in their pure form, and therefore, in practice, stem destruction by the action of a mulcher blade is most often combined.

To implement any model that takes into account the dynamics of the interaction between the working body and the object of impact, as well as the mechanics of destruction itself, certain assumptions must be made.

We shall assume that the stem, as the object of influence, is a cylindrical or thin-walled rod with length L and diameter D, and is described by the Euler–Bernoulli beam model [6].

The stem material is isotropic and has the following parameters: Young's modulus E, density ρ, ultimate normal tensile stress σ_f, and shear stress τ_f.

The mulcher has blades with a mass m that develop an initial speed ϑ_0 and form a contact surface described by the contact area A_c or the radius of the working edge.

In order to theoretically describe the impact interaction of a flexible stem with the working body of a mulcher, it is appropriate to apply the Euler-Bernoulli beam model. Here, we consider the stem as a homogeneous, elastic, isotropic beam with a circular or nearly circular cross-section. We assume that the plant's root system is sufficiently anchored in the soil and that such a beam is conditionally rigidly fixed (clamped) at its base, while the other end of the stem-beam is free. Under these conditions, the Euler-Bernoulli beam model will adequately describe the deformed state of the plant stem under the action of an impulse shock load.

Based on theoretical calculations of Euler-Bernoulli beam theory, the transverse displacements $w(x,t)$ of the beam axis point are determined by the bending differential equation

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}=q(x,t)\ , \qquad (1)$$

E – modulus of elasticity of stem material; I – moment of inertia of a cross-section; ρ – stem material density; A – cross-sectional area; $q(x,t)$ – distributed or concentrated external load, which depends on time.

It is evident that the load $q(x,t)$ at the moment of impact on the stem is impulsive in nature, and the delta function should be used to describe this process. The approximation of such a load will look like this

$$q(x,t)=P_0\delta(x-x_0)\delta(t-t_0)\ , \qquad (2)$$

P_0 – impact force pulse; x_0 – coordinate of the point of application of force; t_0 – moment of impact.

Here, the delta function $\delta(x-x_0)$ indicates that the external force acts only at one point – the point of impact $x=x_0$. In physical terms, this means that $x\in[0,\,l]$ (l – the length of the stem), and the force is applied locally rather than distributed.

For numerical modelling, the delta function $\delta(x-x_0)$ is approximated by ordinary functions with a narrow peak region, such as Gaussian functions

$$\delta_\varepsilon(x-x_0)=\frac{1}{\sqrt{\pi}\varepsilon}\exp\left[-\frac{(x-x_0)^2}{\varepsilon^2}\right], \qquad (3)$$

where $\varepsilon\to 0$.

Contact force can change very quickly over time, and the impact lasts only a moment. Therefore, it is advisable to use a similar time delta function in the analysis.

The contact force will be expressed as

$$P_c=P_0\delta(t-t_0)\ . \qquad (4)$$

Graphical interpretations of the spatial delta function $\delta(x-x_0)$ and temporal delta function $\delta(t-t_0)$ are shown in Fig. 1.

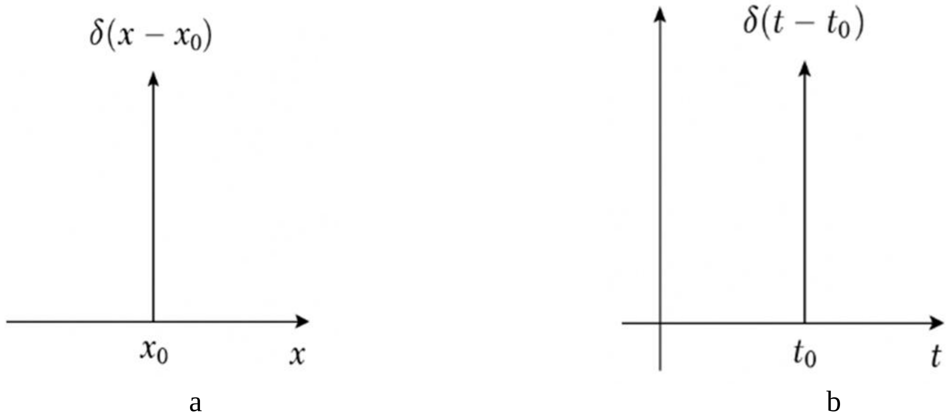

Fig. 1. Graphical interpretations of delta functions: a – spatial delta function; b – temporal

Thus, this approach will make it possible to describe impulse interaction, i.e. when the duration of contact with the stem is significantly less than the stem's own oscillation period. The use of delta functions indicates, however, that the force acts instantaneously at a specific point along the length of the stem and at a specific moment in time, transferring a given impulse to it.

If the conditional beam-stem is fixed at one end and free at the other, the initial conditions will take the form of:

$$w(0,t)=0\ ,\ \frac{\partial w(0,t)}{\partial x}=0\ ,\ \frac{\partial^2 w(l,t)}{\partial x^2}=0\ ,\ \frac{\partial^3 w(l,t)}{\partial x^3}=0\ . \quad (5)$$

After the stem receives an impulse of force, a wave effect occurs in it, spreading along the entire length of the stem. This effect causes the stem to vibrate and deform. The stresses that arise in individual sections of the stem can reach critical values, leading to the destruction of the stem tissue, for example at the point of impact or at the base, where the bending moment will be at its maximum. If this value exceeds the critical value of the bending moment, destruction occurs, i.e.

$$M_{\max} \ge M_{\mathrm{kr}} = \sigma_{\mathrm{kr}} \frac{I}{y_{\max}}\ , \quad (6)$$

σ_{kr} – the bending strength limit of the stem material; $y_{\max}$ – distance from the neutral axis of the outermost fibre of the stem.

This model allows us to evaluate deformations and oscillatory processes after the impact of a blade on a stem. It allows us to determine the distribution of stresses along the length of the stem, as well as the energy that the stem will absorb without breaking. However, the most practical parameter is the determination of the critical speed or impact energy at which the stem is cut or broken. This will allow the structural and kinematic parameters of the rotary working body to be selected for the mulching mode of operation.

We shall construct the solution to the Euler–Bernoulli equation for an impulse impact. We rewrite expression (1) taking into account the load expression (2)

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = P_0\delta(x-x_0)\delta(t-t_0)\ , \quad (7)$$

where P_0 – it is a force impulse defined as $\int F(t)dt = P_0$, and the impact occurs at the initial moment of time $t=0$ at point $x = x_0$.

Then, for the accepted beam model, the following conditions (5) will be valid.

We shall find the eigenfunctions $\{\phi_n(x)\}$ and the eigenvalues $\{\beta_n\}$, which solve the characteristic equation for a beam that is clamped at one end and free at the other.

$$\cosh(\beta_n l)\cos(\beta_n l) = -1\ . \quad (8)$$

The corresponding natural frequencies will be as follows

$$\omega_n^2 = \frac{EI\beta_n^4}{\rho A},\quad n = 1,2,... \quad (9)$$

We shall write the eigenfunctions in non-normalized form

$$\phi_n(x) = \cosh(\beta_n x) - \cos(\beta_n x) - \gamma_n\left(\sinh(\beta_n x) - \sin(\beta_n x)\right), \quad (10)$$

where

$$\gamma_n = \frac{\cosh(\beta_n l) + \cos(\beta_n l)}{\sinh(\beta_n l) - \sin(\beta_n l)}\ .$$

The functions ϕ_n are orthogonal to stem weight ρA

$$\int_0^l \rho A\phi_n(x)\phi_m(m)dx = 0\ ,\ (n \ne m), \quad (11)$$

We shall apply the approach of representing the solution to equation (7) by expanding it in modes

$$w(x,t) = \sum_{n=1}^{\infty} q_n(t)\phi_n(x)\ . \quad (12)$$

We substitute the obtained expression (12) into equation (7) and, utilizing the orthogonality property, we obtain the following for each modal coordinate

$$\ddot{q}_n(t)+\omega_n^2 q_n(t)=\frac{P_0\phi_n(x_0)}{m_n}\delta(t), \tag{13}$$

where mass of the mode

$$m_n=\int_0^l \rho A\phi_n^2(x)dx\ . \tag{14}$$

We assume that the stem was at rest prior to the impact, i.e., $q_n(0^-)=0,\ \dot{q}_n(0^-)=0$. Then, by integrating the mode equation across $t=0$, we obtain the velocity jump

$$\dot{q}_n(0^+)-\dot{q}_n(0^-)=\frac{P_0\phi_n(x_0)}{m_n}\ \text{or}\ \dot{q}_n(0^+)=\frac{P_0\phi_n(x_0)}{m_n}\ . \tag{15}$$

Hence, we conclude that for $t>0$ free oscillations arise, which are described by the expression

$$q_n(t)=\frac{P_0\phi_n(x_0)}{m_n\omega_n}\sin(\omega_n t)\ . \tag{16}$$

Then, expression (12) can be rewritten in the form of

$$w(x,t)=\sum_{n=1}^{\infty}\phi_n(x)\frac{P_0\phi_n(x_0)}{m_n\omega_n}\sin(\omega_n t),\quad t>0. \tag{17}$$

Thus, the expression for the modal expansion for an impulse impact at point x_0 was obtained.

For practical implementation of the task, we understand that P_0 – is not an instantaneous force, but a force impulse. Then, if it is possible to establish the value of the force $F(t)$, which has the duration of contact Δt, then

$$P_0=\int_0^{\Delta t} F(t)dt\ . \tag{18}$$

For numerical calculations, it is convenient to introduce normalised functions

$$\psi_n(x)=\phi_n(x)/\sqrt{m_n}\ , \tag{19}$$

then

$$w(x,t)=P_0\sum_n \psi_n(x)\psi_n(x_0)\frac{\sin(\omega_n t)}{\omega_n}. \tag{20}$$

For real stems, there is energy dissipation, so it is necessary to introduce modal damping in the form of a coefficient ς_n

$$q_n(t)=\frac{P_0\phi_n(x_0)}{m_n\omega_{d,n}}e^{-\varsigma_n\omega_n t}\sin(\omega_{d,n}t)\ , \tag{21}$$

$\omega_{d,n}$ – real frequency (the oscillations are damped),

$$\omega_{d,n}=\omega_n\sqrt{1-\varsigma_n^2}\ . \tag{22}$$

Then the final expression for displacement will be

$$w(x,t)=\sum_n \phi_n(x)\frac{P_0\phi_n(x_0)}{m_n\omega_{d,n}}e^{-\varsigma_n\omega_n t}\sin(\omega_{d,n}t). \tag{23}$$

For a numerical solution of the problem, a truncated series can be considered, but not less than 10 modes for smooth pulses and 50 modes for very short pulses. This number of modes already gives a good approximation of the displacement and depends, in turn, on the geometry and parameters of the interacting bodies.

Upon determining the displacement (23), it becomes possible to establish the curvature $\frac{\partial^2 w(x,t)}{\partial x^2}$ and the moment

$$M(x,t)=-EI\frac{\partial^2 w(x,t)}{\partial x^2}\ . \tag{24}$$

If we compare the maximum moment found $\left|M(x,t)\right|$ with the critical value M_{kr}, we obtain a failure criterion that allows us to predict stem failure.

After presenting the main material, we shall return to the practical objective of the work–determining the peripheral speed of the working element's blade interaction with the stem required for its breakage. This parameter is necessary for the technical implementation of the rotary working element.

In order to establish the minimum required rotational speed of the shaft of the rotating working body based on the developed theoretical model of interaction, we will conduct a numerical experiment. We adopt the following parameters: rotor radius $R = 0.3$ m; number of blades around the drum $z = 4$; peripheral speed range 6…36 m/s; we shall adopt potato haulm as the primary plant mass: average stem diameter $D = 8$ mm; plant height above the field surface $L = 0.5$ m; Young's modulus of the stem $E = 12$ MPa (average value); ultimate bending strength $\sigma_{\text{kr}} = 5$ MPa; the density of the plant mass is approximately $\rho = 1000$ kg/m^3; the working body – Y-shaped blade: mass $m = 0.2$ kg; the stem damping coefficient is $\varsigma_n = 0.03$ [3, 9].

To determine the required speed of interaction between the working body and the stem, we use the Euler-Bernoulli beam model. The result is obtained by comparing the maximum moment acting on the stem, which depends on the speed of interaction, and comparing it with the critical breaking moment. For process reliability, we introduce a safety factor for bending and set a 20% margin for process reliability

$$s_{\vartheta} = \frac{M_{\max}}{M_{kr}} \geq 1,2\ . \tag{25}$$

Using a numerical method in the Mathcad 14 application to find the value $M_{\max}$ as a function of the circular speed of the flail and taking into account the accepted initial parameter values, we obtain the range of circular speeds of the blade at which the stems of the plants being removed are destroyed. We will show a graphical interpretation of the results obtained using the Excel application package, Fig. 2.

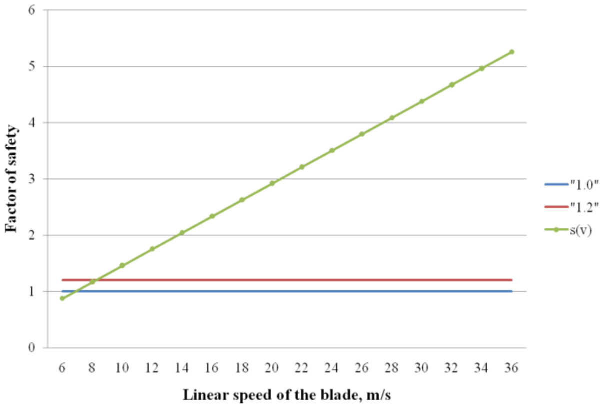

Fig. 2. Dependence of the change in the safety factor for the bending condition on the peripheral speed of the blade

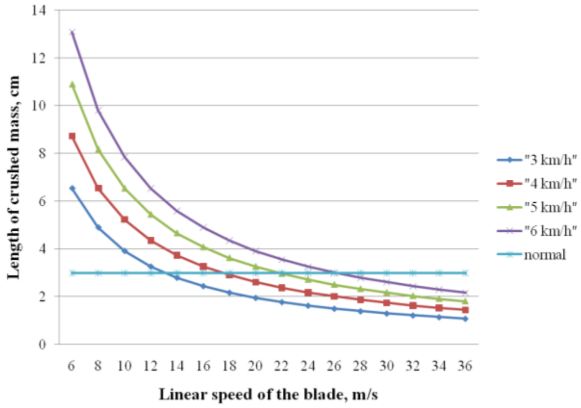

Fig. 3. Dependence of the shredded material length on the peripheral speed of the blade and the forward speed of the unit

5. Conclusion

The results of the numerical experiment indicate that the following parameters of the rotary working body are valid for the accepted initial data. At a rotor speed of 255 rpm, which corresponds to a circular speed of the blade of 8 m/s, the safety factor for bending (25) is 1.17. The condition is not met, but the value is close to meeting it. At this frequency, stem breakage is likely to occur, since the bending moment created by the blade exceeds the critical moment by 1.17 times. The specified rotor rotation speed can be used mainly when working with large-stemmed plants and only for breaking them down. Removal by shredding will be practically impossible here. By increasing the rotor speed, the blades will have a higher circumferential speed and, starting from 10 m/s (318 rpm), condition (25) will be reliably fulfilled. The stems will be broken by the flails. However, in order to effectively remove 'excess' vegetation from the surface of the field or between rows of row crops, each stem must be struck multiple times by a blade. For this purpose, it was proposed to install four blades in the circumferential direction of the drum and set the shredded material length, for example ≤ 3 cm. Applying the above criteria, the following calculation results were obtained (Fig. 3). The length of the shredded material of approximately 3 cm will be achieved at a peripheral speed of the blade of 12–14 m/s at a forward speed of the unit of 3.0 km/h (0.83 m/s); for a forward speed of the unit of 4.0 km/h (1.11 m/s) – 16–18 m/s; for 5.0 km/h (1.39 m/s) – 20–22 m/s; for 6.0 km/h (1.67 m/s) – 24–28 m/s.

Thus, the implementation of the mathematical model for the interaction between the blade of the rotary working element and the stems of the plants being removed, based on the Euler–Bernoulli beam model, allowed us to substantiate the minimum peripheral speed of the working element for various forward speeds of the machine and the required length of the shredded material.

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МАТЕМАТИЧНА МОДЕЛЬ КОНТАКТНОЇ ВЗАЄМОДІЇ РОТАЦІЙНОГО РОБОЧОГО ОРГАНУ ЗІ СТЕБЛАМИ РОСЛИН ДЛЯ ЇХ ПОДРІБНЕННЯ

У роботі запропонована математична модель контактної взаємодії била як робочого органу ротаційного подрібнювача рослинних решток зі стеблом рослин, що видаляються з поверхні поля. Спосіб видалення такої рослинності передбачає їх подрібнення та розкидання по поверхні поля. Практичною метою реалізації математичної моделі є визначення мінімальної колової швидкості робочого органу при його взаємодії зі стеблом.

Математична модель базується на припущені, що стебло рослини можна моделювати, застосовуючи теорію балки Ейлера-Бернулі. Для теоретичного відтворення ударної імпульсної взаємодії била по стеблу використано апроксимацію навантаження із застосуванням дельта-функцій, що мають вузьку пікову ділянку. Наведена модель дозволяє оцінити деформації та коливні процеси, що виникають у стеблі після удару билом ротаційного робочого органу. Побудовано розв'язок рівняння Ейлера-Бернулі для імпульсного удару. Отримані вирази поперечних переміщень точки осі балки мають модальний розклад для імпульсного удару. Оскільки реальні стебла мають фактичне розсіювання енергії, то у модель введено модальний демпінг, що виражений відповідним коефіцієнтом. Для чисельного розв'язку поставленої задачі розглянуто усічений ряд: 10 мод для гладких імпульсів і 50 мод для дуже коротких імпульсів. Така кількість мод дає вже добру апроксимацію переміщення та залежить від геометрії та параметрів взаємодіючих тіл.

Результатом числового експерименту є отримання значень максимальних згинних моментів, що викликають напруження згину та проведено їх порівняння з критичними напруженнями, що викликають руйнування стебла. Для надійності отриманих результатів коефіцієнт запасу міцності за умовою згину збільшено до 1,2, що буде забезпечувати гарантований злом стебла. На основі цього знайдено мінімальну швидкість взаємодії била зі стеблом при виконанні умови зламу. Така швидкість складає від 8-10 м/с, при цьому стебла будуть тільки переламаними, але не подрібненими. Наклавши умову подрібнення стебел на частинки довжиною приблизно 0,03 м, отримано ряд колових швидкостей била ротаційного робочого органу для варіантів поступальної швидкості агрегату. Наприклад, при поступальній швидкості агрегату 3,0 км/год (0,83 м/с) колова швидкість била повинна складати 12-14 м/с; для поступальної швидкості агрегату 4,0 км/год (1,11 м/с) – 16-18 м/с; для 5,0 км/год (1,39 м/с) – 20-22 м/с; для 6,0 км/год (1,67 м/с) – 24-28 м/с. Таким чином, реалізація математичної моделі взаємодії била ротаційного робочого органу із стеблом рослин, що видаляються, на основі моделі балки Ейлера-Бернулі дозволило нам обґрунтувати мінімальну колову швидкість робочого органу для різних поступальних швидкостей агрегату та необхідної довжини подрібненої маси.

Ключові слова: *стебло, подрібнення, мульчувач, збирання, згин, напруження, момент, швидкість, ротаційний робочий орган, математична модель, удар, енергія, експеримент, процес, руйнування, зрізування.*

Ф. 25. Рис. 3. Літ. 17.

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