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**PERFECTION OF FUNDAMENTALS OF MATHEMATICAL METHOD OF DESIGN OF
HYDROSYSTEMS OF DRIVE OF TECHNICAL MACHINES****Svetlana Kravets**, Assistant
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The vast majority of modern research on the study of hydromechanical systems is based on the use of the theory of automatic control and management. One of the main components of technical machines and cells are their drives. Hydraulic drives are often used as power. In general, its functional and operational characteristics have a decisive influence on the properties of hydrodynamic systems and general technical machines. In this regard, close attention is paid to the study of hydraulic power drives. However, the power hydraulic drive of technical machines and units in one form or another necessarily includes both electric and mechanical drives, which turns it into a system of various drive systems. The vast majority of modern research is devoted to the study of hydraulic drives, which are based on spool-type hydraulics, which limits the possibility of using new structural materials in the drive systems of technical machines.

Many technical machines and installations, especially in agro-industrial complexes, work in harsh conditions, with high temperatures and dust, low quality of working fluid and other unfavorable characteristics that significantly reduce the reliability of drive systems equipped with hydraulic spool types and quality equipment. At current levels of working pressure in the hydraulic system, the forces and speeds of the working bodies of technical machines and units of the agro-industrial complex, the compressibility of the working fluid and the flexibility of the hydraulic elements of the drive system are determined, become noticeable, which requires further improvement of theory and methodology.

The theoretical basis of the method of calculating the equipment drive system has been improved on the basis of the complex characteristics of the hydraulic system introduced by the author - the resulting volumetric stiffness is used to simplify the drive modeling process by increasing the accuracy of calculation and analysis of its results.

Key words: method, hydraulic drive, volume rigidity, modeling, drive, system, machine.

F. 14. Fig. 3. Ref. 13.

1. Formulation of the problem

As shown by the results of the analysis of the properties of drive systems of process equipment [1], one of the main advantages of hydraulic drives is their rigidity, and it should be noted that the rigidity of hydraulic systems of drives of process equipment is higher than that of other types of drives (electric, pneumatic and magnetic).

In addition, the rigidity can vary depending on the requirements for them. At the same time, methods for calculating the rigidity of hydraulic systems of drives of technological equipment are of interest.

2. Analysis of recent research and publications

Main part. The vast majority of modern research devoted to the study of hydromechanical systems is based on the use of the theory of automatic regulation and control. The mathematical model of the hydraulic system in this case is compiled on the basis of the flow continuity equation, while taking into account the compressibility of the liquid and the compliance of the pipelines is carried out by introducing into the equation a certain fictitious flow rate of the liquid spent on changing the volume of the working fluid during its compression (expansion) and changing the internal volume of the pipeline section during its expansion (compression), the values of which depend on the reduced modulus of elasticity of the pipeline [2].

Known methods for calculating the power hydraulic drive and other hydraulic systems, taking into account the violation of the equation of continuity of the flow during the operation of the system in unsteady modes. Thus, in [3, 4, 5] a numerical description of hydraulic drives is proposed, in which the “reduced



stiffness of the pipeline element" [4] is used to determine the pressure increment in the hydraulic lines, and in the hydraulic cylinder the "stiffness of the cavity under the piston face" [3].

3. The aim of the study

To analyze the improvement of scientific and methodological basis design of systems of drives of technological machines.

4. Presenting main material

In order to increase the versatility of this method of calculating hydraulic drives and streamline it, we propose to introduce the concept of "volumetric stiffness of hydraulic systems" into the practice of calculations as an integrative criterion for assessing their dynamic properties [6].

The essence of the concept of "volumetric stiffness" can be revealed by referring to the generalized Hooke's law, from which it follows that the change in fluid pressure at various points of the hydraulic system is proportional to the change in its initial volume [7]:

$$dp = -E_l \frac{dV_l}{V_0},$$

then, denoting

$$C_l = \frac{E_l}{V_0}, \quad (1)$$

we get

$$dp = -C_l dV_l,$$

where

$$C_l = -E_l \frac{dp}{dV_l}, \quad (2)$$

where V_0 – is the initial volume of liquid; dV_l – is the increment in the volume of liquid corresponding to the increment in pressure by dp ; $E_l = \frac{dp}{dV_l}$ – bulk modulus of elasticity of the liquid; β_l – liquid volumetric compression ratio; C_l – coefficient of "volumetric rigidity" of a certain volume of fluid, which from equation (1) can be determined as the ratio of the bulk modulus of elasticity of the working fluid to the volume occupied by it. Thus, "volumetric stiffness" characterizes the stiffness of some (well-defined) volume of liquid.

Equation (2) shows that the "volumetric stiffness" of a fluid is the increment in fluid pressure corresponding to a unit increment of its initial volume that occurred under the action of this increment of pressure, or, in other words, the "volumetric stiffness" of a fluid is the increment in pressure necessary to change the initial volume of liquid per unit. The minus sign in (2) indicates that a positive pressure increment dp corresponds to a negative fluid volume increment dV_l and vice versa.

Equation (1) makes it possible to determine the volumetric stiffness of a working fluid bounded by an absolutely rigid shell. However, in real hydraulic drives of technological equipment, a liquid (elastic) working medium moves through pipelines and other elements of the system subject to volumetric deformation, which must be taken into account when designing hydraulic drives with desired properties. To this end, we will expand the concept of "volumetric stiffness" to the entire hydraulic system by introducing the concept of "reduced volumetric stiffness" of the hydraulic system and its elements.

"Reduced volumetric rigidity" allows one to judge the compliance of the system as a whole and each of its elements separately, similarly to the concept of rigidity of solids, which indicates the property of solids to resist changing the linear dimensions of these bodies when external forces act on them. In this case, the "reduced volumetric stiffness" makes it possible to estimate the spatial deformation, which is necessary to describe the compliance of liquid media and their shells. According to the described logic, we write an equation for calculating the coefficient of "reduced volumetric stiffness" of the hydraulic system for driving technological equipment [1]:

$$C_r = \frac{dp}{dV_l}, \quad (3)$$

where C_r – reduced volumetric stiffness of the considered section of the hydraulic drive (its element) or the system as a whole; dp – is the pressure increment in the considered section of the hydraulic drive (on its element) or in the system as a whole; dV – is the increase in the internal volume of the considered section of the hydraulic drive (its element) or the system as a whole, corresponding to the increase in pressure dp .

Equation (3) allows us to define the reduced volumetric stiffness of the hydraulic drive system or its element [2].



The volumetric stiffness of the hydraulic system for driving technological equipment is the property of the hydraulic system to resist changes in the internal volume under pressure.

Considering that the change in the volume of the hydraulic system occurs as a result of an increase in the pressure of the liquid, which itself is compressed at the same time, the definition for the “reduced volumetric stiffness coefficient” of the hydraulic system of the drive of technological equipment will be written as follows: the coefficient of the reduced volumetric stiffness of the hydraulic system of the drive of technological equipment is equal to the increase in pressure, which occurs due to the introduction into it of an additional unit volume of the working fluid, reduced to the pressure of the external environment.

The use of the coefficient of reduced volumetric stiffness C_r makes it possible to describe the system both when the fluid is stationary and when it moves, in the latter case, the equation [3] is used:

$$dp = C_r(\Sigma Q_{entry} - \Sigma Q_{exit}) \cdot dt, \quad (4)$$

where dp – is the fluid pressure increment in the considered volume of the hydraulic system during the time dt ; ΣQ_{entry} і ΣQ_{exit} – are the total flow rates of the fluid entering and exiting the volume under consideration, respectively; C_r – is the coefficient of the reduced volumetric stiffness of the hydraulic system or its element.

Consider the case when n different elements are connected to a certain point of the hydraulic system, each of which has its own volumetric rigidity. Then, assuming that the change in pressure at a given point is instantly transmitted to all points of the considered volume, we write:

$$dV = \sum_{i=1}^n dV_i,$$

where dV – is the volume of liquid, reduced to ambient pressure, required to change the pressure in the system by dp ; dV_i is the volume of liquid reduced to the ambient pressure required to change the pressure in the i -th element by dp .

The coefficient of the reduced volumetric stiffness of the system as a whole is determined by the equation [4]:

$$C_r = \frac{dp}{\sum_{i=1}^n dV_i} = \frac{1}{\sum_{i=1}^n \frac{dV_i}{dp}} = \frac{1}{\sum_{i=1}^n \frac{1}{C_i}}$$

after transformations, we finally get:

$$C_r = \frac{\prod_{i=1}^n C_i}{\sum_{i=1}^n \left(\frac{1}{C_i} \prod_{i=1}^n C_i\right)}, \quad (5)$$

where C_r is the coefficient of the reduced volumetric stiffness of the hydraulic system, consisting of n jointly working elements; C_i - is the coefficient of volumetric rigidity of the i -th element of the system.

Coefficient of volumetric rigidity of hydraulic lines (pipelines). It is obvious that when the pressure inside the pipeline changes as a result of deformation, its internal volume will change, which is determined by the equation [5]:

$$V_p = \pi \cdot r^2 \cdot l,$$

where r and l are the current values of the inner radius and length of the pipe section under consideration, respectively.

The change in the volume of the pipeline with a change in pressure can be estimated by knowing the coefficient of its volumetric rigidity, found by equation (3), in which the volume increment is determined by the formula [6]:

$$dV_p = 2\pi \cdot r \cdot l \cdot dr + \pi \cdot r^2 \cdot dl, \quad (6)$$

where dr and dl are the increment of the pipe radius and its length corresponding to the pressure increment on dp .

At the same time, it is known that

$$dr = \frac{r}{E_{p.w.m.}} d\sigma \quad (7)$$

where $E_{p.w.m.}$ - is the modulus of elasticity of the pipe wall material; $d\sigma$ - is the increment of stress that occurs in the pipe wall, corresponding to a change in pressure by dp . The stress that occurs in the pipe wall when the liquid pressure p - is applied to it from the inside can be determined by the formula [7]:

$$\sigma = p \frac{r}{\delta},$$

where δ - is the pipe wall thickness.



Then the total stress differential in the wall material is found by the expression:

$$d\sigma = p \frac{1}{\delta} (rdp + pdr), \quad (8)$$

but since for pipelines with an elastic modulus significantly exceeding the elastic modulus of the fluid flowing through them ($E_{p.w.m.} \gg E_l$), we can assume that

$$rdp \gg pdr, \quad (9)$$

then, neglecting the second term in the bracket of the total voltage differential (8), equation (7) can be written as:

$$dr = \frac{1}{\delta} \frac{r^2}{E_{p.w.m.}} dp, \quad (10)$$

At the same time, we note that, in accordance with Hooke's law, the increment in the pipeline length under the action of a tensile force is determined by the formula [8]:

$$dl = \pi \cdot r^2 \cdot dp \cdot \frac{l}{E_{p.w.m.} \cdot \pi \cdot (r_c^2 - r^2)}, \quad (11)$$

where l is the current value of the pipeline length; r_c is the current value of the outer radius of the pipeline.

Substituting the values of dr (10) and dl (11) into equation (6), we make transformations and obtain:

$$dV_p = \frac{\pi r^2 l}{E_{p.w.m.}} \left(\frac{2r}{\delta} + \frac{r^2}{r_c^2 - r^2} \right) dp$$

Or, replacing the radii with the corresponding diameters, we get:

$$dV_p = \frac{\pi d_{p.w.m}^2 l}{4E_{p.w.m.}} \left(\frac{2}{d_p - l} + \frac{1}{d_p^2 - l} \right) dp \quad (12)$$

Where $d_p = \frac{d_c}{d_p}$ is the ratio of the current value of the outer diameter of a cylindrical pipeline to the current value of its inner diameter [9].

Then from equation (3), taking into account (12), we finally obtain an expression for determining coefficient of volumetric stiffness of the pipe section wall:

$$C_{r.v.p.} = \frac{4E_{p.w.m.}}{\pi d_{p.w.m}^2 l} \frac{l}{\left(\frac{2}{d_p - l} + \frac{1}{d_p^2 - l} \right)}$$

Obviously, generally, the coefficient of volumetric rigidity of the pipeline wall (13) is a non-linear value that depends on the current values of the pipeline diameters and its length, which are determined by pressure [9], the change of which is determined by the volumetric rigidity of the pipeline wall. Generally, the coefficient of the reduced volumetric stiffness of the pipeline with a working liquid is determined by equation (5) taking into account (1) and (13):

$$C_{r.v.p.} = \frac{4E_{p.w.m.}}{\pi d_{p.w.m}^2 l} \frac{l}{1 + \frac{E_l}{E_{p.w.m.}} \left(\frac{2}{d_p - l} + \frac{1}{d_p^2 - l} \right)} \quad (13)$$

If, however, the deformation of the pipeline in the axial direction (pipeline with a small length and diameter) and the nonlinearity of the coefficient of reduced volumetric rigidity pipeline can be neglected, it can be determined by the equation [10]:

$$C_{r.v.p.} = \frac{E_l}{V_p \left(1 + \frac{d_0}{\delta} \frac{E_l}{E_{p.w.m.}} \right)} \quad (14)$$

where $C_{r.v.p.}$ is the coefficient of the reduced volumetric stiffness of the pipeline with the working fluid; V_p - the value of the internal volume of the pipeline before its deformation; d_0 and δ are the initial the value of the inner diameter and the wall thickness of the pipeline (taken constant) respectively.

Determining the coefficient of the reduced volumetric stiffness of the cylindrical wall by equations (13) and (14), one should keep in mind condition (9), from which it follows that they can be used only for the calculation of cylindrical pipes made of metal or other material with a sufficiently large modulus of elasticity. At the same time, in the power hydraulic drive of various equipment, high-pressure flexible hoses (HPR) are widely used, which are rubber-cord a shell whose elastic properties are not amenable to a rigorous analytical description [11]. At In this case, it is obvious that the reduced volumetric stiffness of such shells has a dependence on pressure, which should be determined experimentally. Figures 1, 2, 3 show the experimentally obtained [12] graphs of the pressure dependence of the coefficients of the reduced volumetric stiffness of various high pressure hoses.

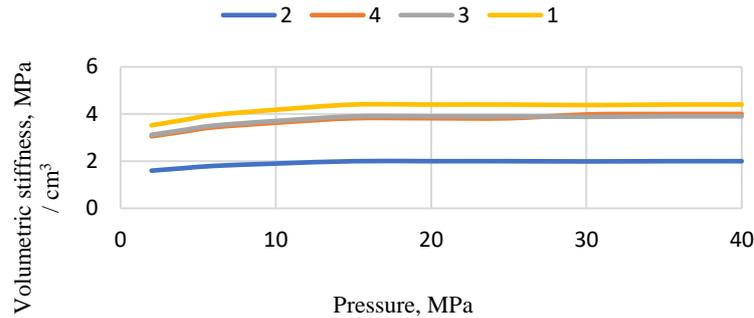


Fig. 1. Graphs of the dependence of the reduced volumetric stiffness coefficients on pressure: 1 – pressure dependence of the coefficient of reduced volumetric stiffness of a high pressure hose with a nominal diameter $d_1 = 16$ mm, the length of which, without taking into account the length of the fitting nipples (net length of the rubber-cord sheath), is $L_{11} = 0.94$ m; 2 – pressure dependence of the coefficient of reduced volumetric stiffness of the same high pressure hose, the length of which, without taking into account the length of the fitting nipples, is $L_{12} = 1.94$ m; 3, 4 – pressure dependences of the coefficients of the reduced volumetric stiffness of the same hoses, recalculated according to equation (5) per 1 meter of their length

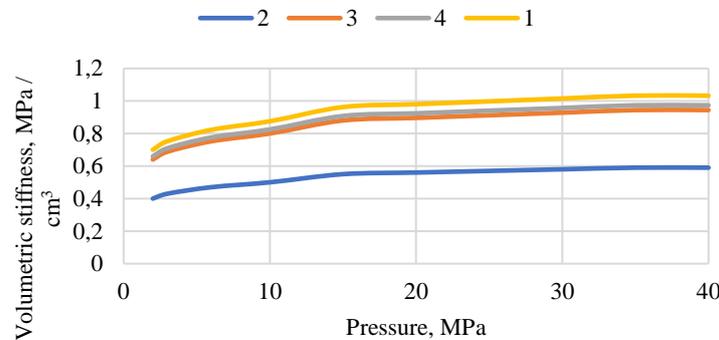


Fig.2. Graphs of the dependence of the coefficients of reduced volumetric stiffness on pressure: 1 – pressure dependence of the coefficient of reduced volumetric stiffness of a high pressure hose with a nominal diameter $d_2 = 32$ mm, the length of which, without taking into account the length of the fitting nipples, is $L_{21} = 0.88$ m; 2 – pressure dependence of the coefficient of reduced volumetric stiffness of the same high pressure hose, length which, without taking into account the length of the fitting nipples, is $L_{22} = 1.88$ m; 3, 4 – pressure dependences of the coefficients of the reduced volumetric stiffness of the same hoses, recalculated according to equation (5) per 1 linear meter of their length

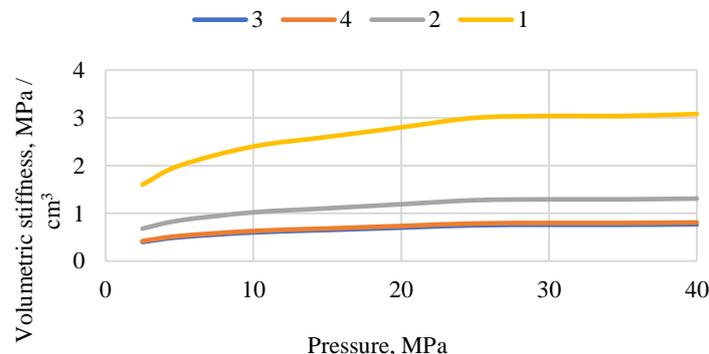


Fig.3. Graphs of the dependence of the coefficients of the reduced volumetric stiffness of the high pressure hose on pressure when they are included in the circuit in series: 1 - pressure dependence of the coefficient of reduced volumetric stiffness of high pressure hoses with a nominal diameter $d_1 = 16$ mm, the net length of the rubber-cord shell of which is $L_{12} = 1.94$; 2 - pressure dependence of the coefficient of the reduced volumetric stiffness of the high pressure hose with a nominal diameter $d_2 = 32$ mm, the net length of the rubber cord shell of which is $L_{21} = 0.88$; 3, 4 - pressure dependences of the coefficients reduced volumetric stiffness of the above sleeves connected in parallel in the circuit; (3 - determined by equation (5) using experimental data)



Analyzing the graphs shown in Fig. 1, 2, 3, it is easy to see that the coefficients reduced volumetric stiffness of all investigated high-pressure hoses in the zone of low pressure ($p < 15$ MPa) increase non-linearly. This character of the change in the coefficient of reduced volumetric stiffness can be explained by the fact that in this zone of pressure change the shell expands due to the selection of gaps between the strings of a braided metal cord, as well as possible presence of undissolved air in the system. With further increase in pressure working fluid inside the shells, the coefficients of their reduced volumetric stiffness change linearly, since in this case their volume changes due to the stretching of the strings forming cord, and changes in the modulus of elasticity of the liquid itself [13]. That is, here, with some approximation, the HPH can be considered as a metal shell with certain rigidity characteristics.

Obviously, to describe the stiffness characteristics of high pressure hoses of different lengths, there is no need to experimentally determine the coefficient of the reduced volumetric stiffness of each of them. It is sufficient to know the coefficient of reduced volumetric stiffness per unit length of each type of shell, which will allow, using Eq. (5), to calculate the coefficient of reduced volumetric stiffness of a high-pressure hose of any length.

5. Conclusions

Improving the methods for calculating and designing drive systems for technological equipment based on the application of the reduced volume stiffness coefficient hydraulic system during their modeling allows to simplify the modeling process along with the simultaneous increase in the accuracy of calculations and the quality (visibility) of their analysis.

References

- [1] Ratushna, N., Mahmudov, I., Kokhno A. (2007). Metodichni pidkhody do stvorennia novoi silskokhospodarskoi tekhniki u vidpovidnosti z vymohamy rynku naukoiemnoi produktsii. *MOTROL*. 9A. 119–123. [in Ukrainian].
- [2] Hun'ko I.V., Burlaka S.A., Yelenych A.P. (2018) Otsinka ekolohichnosti naftovoho palyva ta biopalyva z vykorystanniam metodolohiyi povnoho zhyttyevoho tsyклу. *Visnyk Khmel'nyts'koho natsional'noho universytetu*. Tom 2. 6 (267). S. 246–249. [In Ukrainian].
- [3] Lupenka, Yu. O., Mesel-Veseliaka, V.Ya. (2012). Stratehichni napriamy rozvytku silskoho gospodarstva Ukrainy na period do 2020 roku. K. : NNTS “IAE”. [in Ukrainian].
- [4] Okocha A. I., Antypenko A. M. (1996) Palyvno-mastylni ta inshi ekspluatatsiyni materialy. K.: Urozhay, 336 s. [In Ukrainian].
- [5] Muzychuk V.I., Nakhaychuk O.V., Komakha V.P. (2012) Vyznachennya zmistu i ob'yemu robit pry tekhnichnomu servisi. *Zb.nauk.pr. VNAU. Seriya: Tekhnichni nauky*. Vinnytsya: VNAU, № 11 (65). S. 242–247. [In Ukrainian].
- [6] Muzychuk V.I., Anisimov V. F. (2012) Orhanizatsiya robit pidpryyemstv tekhnichnoho obsluhovuvannya: navchal'nyy posibnyk. Vinnytsya: FOP Rohal's'ka I.O., 240 s. [In Ukrainian].
- [7] Ivanov, N., Sharhorodskyy, S., Rutkevych, V. (2013). Matematicheskaia model hidropivoda blochno-portsionoho otdelitelia konservirovannykh kormov. *MOTROL*. 5. 83–91.
- [8] Pacstyushenko, S.I. (2002). Pytannia optymizatsii tekhnichnykh system. *Zbirnyk naukovykh prats NAU “Mekhanizatsiia silskohospodarskoho vyrobnytstva”*. Kyiv: Vydavnytstvo NAU. T.XI. 266–271. [in Ukrainian].
- [9] Hryhor'yev M.A., Ponomarev N. N., Karpenko V. V. (1979) Metodyka otsinky resursu dvyhuna zalezno vid resursiv yoho detaley. M.: Avtomobil'na promyslovist', 10, S.4–6. [In Ukrainian].
- [10] Anisimov V. F., Yatskovs'kyi V. I., P'yasets'kyi A. A. Ryaboshapka V. B. (2011) Napryamky stvorennia bahatopalyvnykh dvyhuniv na bazi dyzel'noho tsyклу. *Promyslova hidravlika i pnevmatyka*. №2 (32). C. 100-105. [In Ukrainian].
- [11] Burlaka S.A., Yavdyk V.V., Yelenych A.P. (2019) Metody doslidzhen' ta sposoby otsinky vplyvu palyv z vidnovlyuvanykh resursiv na robotu dyzel'noho dvyhuna. *Visnyk Khmel'nyts'koho natsional'noho universytetu*. 2 (271). S. 212–220. [In Ukrainian].
- [12] Malakov O.I., Burlaka S.A., Mykhal'ova Y.O. (2019) Matematychni modelyuvannya ta osnovy konstruyuvannya vibratsiynykh zmishuvachiv. *Visnyk Khmel'nyts'koho natsional'noho universytetu*. 5 (277). S. 30-33. [In Ukrainian].
- [13] Ivanov, M.I., Sharhorodskyy, S.A., Rutkevych, V.S. (2013). Pidvyshchennia ekspluatatsiinoi efektyvnosti blochno-portsiinoho vyvantazhuvacha konservovanykh kormiv shliakhom hidrofikatsii



pryvoda robochikh orhaniv [Improving the operational efficiency of the block-batch unloader of canned fodder by hydration of the drive of working bodies.]. Promyslova hidravlika i pnevmatyka – Industrial hydraulics and pneumatics. 1(39). 91–96 [in Ukrainian].

ДОСКОНАЛІСТЬ ОСНОВ МАТЕМАТИЧНОГО МЕТОДУ ПРОЕКТУВАННЯ ГІДРОСИСТЕМ ПРИВОДУ ТЕХНІЧНИХ МАШИН

Переважає більшість сучасних досліджень, присвячених вивченню гідромеханічних систем, ґрунтується на використанні теорії автоматичного регулювання та управління. Однією з основних складових технічних машин і осередків є їх приводи. Гідравлічні приводи часто використовуються як силові. Загалом, його функціональні та експлуатаційні характеристики мають вирішальний вплив на властивості гідродинамічних систем і загальнотехнічних машин. У зв'язку з цим пильну увагу приділено дослідженням гідравлічних силових приводів. Однак силовий гідропривод технічних машин і агрегатів в тій чи іншій формі обов'язково включає як електричний, так і механічний приводи, що перетворює його в систему різноманітних систем приводу. Переважає більшість сучасних досліджень присвячена вивченню гідроприводів, в основі яких лежить гідравліка золотникового типу, що обмежує можливість використання нових конструкційних матеріалів у системах приводів технічних машин.

Багато технічних машин і установок, особливо в агропромислових комплексах, працюють в жорстких умовах, з високими температурами і запиленістю, низькою якістю робочої рідини та іншими несприятливими характеристиками, що значно знижують надійність систем приводу, оснащених гідравлічними золотниковими типами і якісним обладнанням. При поточних рівнях робочого тиску в гідросистемі визначаються сили і швидкості руху робочих органів технічних машин і агрегатів АПК, стисливість робочої рідини і гнучкість гідравлічних елементів приводної системи. стають помітними, що вимагає подальшого вдосконалення теорії та методики.

Теоретична основа методики розрахунку системи приводу обладнання вдосконалена на основі введених автором комплексних характеристик гідравлічної системи - використана результуюча об'ємна жорсткість дозволяє спростити процес моделювання приводу, підвищивши точність розрахунку та аналізу його результатів.

Ключові слова: метод, гідропривід, об'ємна жорсткість, моделювання, привід, система, машина.

Ф. 14. Рис. 3. Літ. 13.

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