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APPLICATION OF ENVELOPED DIAGRAMS OF STATOR CURRENTS IN THE PROBLEMS OF DIAGNOSING BEARINGS OF INDUCTION MOTORS

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Today, in conditions of given the high demands for reliability and uninterrupted operation of equipment, electric machines are used in important and even critical technological processes, where their failure can lead to serious consequences, including production downtime, material losses, or even safety hazards. Diagnostic systems enable the timely detection of potential faults, which helps prevent emergency situations and significantly reduces costs associated with unforeseen repairs.

Moreover, with the advancement of technology, the design of electric machines is becoming increasingly complex, necessitating the implementation of more precise monitoring methods. Such diagnostics not only help prevent emergency situations but also extend the service life of equipment. Additionally, the timely detection of defects helps maintain high energy efficiency in machines, which is crucial for reducing energy costs and minimizing the negative environmental impact.

Integrating diagnostic systems into the overall automated infrastructure allows for real-time monitoring and more accurate forecasting of potential failures. These approaches reduce the risk of accidents, improve the efficiency of production process management, and ensure safety. In the context of the transition to Industry 4.0 and the widespread use of digital technologies, such systems have become an indispensable element for ensuring the stable and safe operation of rotating electric machines, which is critical for modern industrial enterprises. They also allow for a significant reduction in the energy intensity of production processes by extending the operational life of primary production assets.

The work proposes diagnostic indexes for detecting bearing defects in induction motors, theoretically justifying the use of the Hilbert transform for current-based bearing defect diagnostics, and demonstrating that the spectrum of the current envelope waveform should be plotted on a linear scale.

Key words: *diagnostics, induction motor, defect, bearing assembly, emergency failure, envelope of current diagram.*

Eq. 9. Fig. 1. Ref. 10.

1. Problem formulation

The analysis of envelope waveforms holds significant practical value in the theory of signal processing. The processing of envelope waveforms of currents, voltages, and torques is also applied in the analysis of transient processes in powerful rotating machines. This method is particularly important in the diagnostics of machines, especially when analyzing the spectra of envelope vibration signals for the early detection of machine defects [4].

In classical spectral analysis, the original current waveforms are used, which allow for diagnosing machine faults [5]. A fault in the motor leads to periodic amplitude modulation of the current, causing a change in the amplitude of its waveform [6].

Based on existing research results and theoretical considerations, it can be assumed that the resulting curve of current waveform amplitudes for a healthy motor is approximated by a straight line, while for a faulty motor, it corresponds to distinct oscillations (pulsations). In several studies, the analysis of current envelope waveforms is used for diagnosing rotor faults [7]. This research aims to develop a method and mathematical framework for detecting bearing defects based on the analysis of the envelope diagram of the stator current of an induction motor.

2. Analysis of recent research and publications

Induction motors (IM) play a crucial role in modern industry. They are widely used across various sectors due to their versatility and ability to operate in challenging conditions, making them indispensable for many manufacturing processes. Furthermore, the ability to regulate speed using frequency converters allows them to be easily integrated into automated systems, providing precise control over processes [1].

These motors are known for their high mechanical strength, which enables them to handle heavy loads, and the absence of moving parts like brushes contributes to their longevity and reduces maintenance costs. All these characteristics make induction motors essential for the efficient operation of modern manufacturing, ensuring stability, cost-effectiveness, and reliability.

Another trend in post-industrial production is the reduction of the energy intensity of manufacturing processes, which is also manifested by extending the operational lifespan of power equipment until its complete physical wear, as the latter is characterized by relatively slow amortization of the second type and high resource consumption [2, 3]. Since the implementation of such a resource management strategy requires continuous monitoring of the equipment's technical condition, it is clear that the development of automated technical diagnostic systems for induction electric machines holds significant theoretical and practical value.

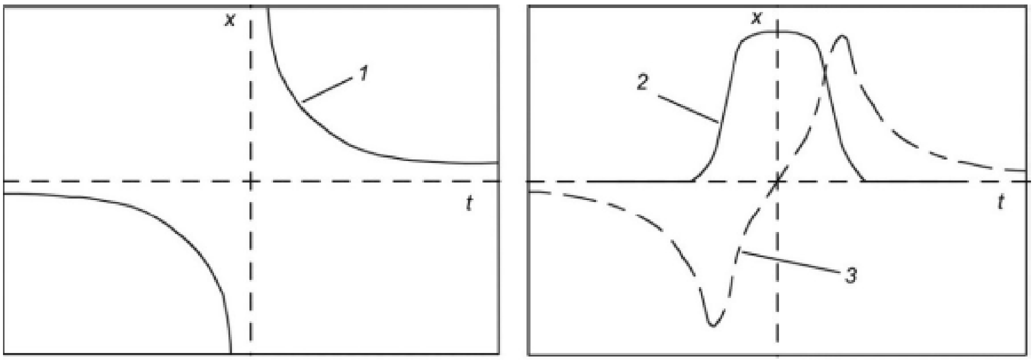
3. The purpose of the article

The purpose of this work is to develop a method and mathematical apparatus for diagnosing defects in the bearing assembly of an asynchronous electric motor based on the analysis of the stator current envelope diagram. Special attention is paid to substantiating the feasibility of using the Hilbert transform and constructing the current envelope spectrum on a linear scale as an effective diagnostic index. The proposed approaches are aimed at increasing the accuracy of diagnostics, ensuring reliable and safe operation of electric machines in industrial conditions, as well as integrating diagnostic systems into the general infrastructure of automated control

4. Results and discussion

Current fluctuations due to bearing wear are found in the low-frequency and mid-frequency ranges of the spectrum [6, 8]. Therefore, it is advisable to obtain the envelopes of low-frequency and mid-frequency oscillations by applying the Hilbert transform.

The Hilbert transform performs a 90° phase shift (π/2 shift) on all frequency components of the signal, while preserving the amplitude relationships in the signal spectrum, removing only the constant component [9]. In the current spectrum based on the envelope waveform, the dominant harmonic is the frequency f_0 = 0 Hz, which is related to the fact that the Hilbert operator has a function of this form $h(t)=\frac{1}{\pi t}$. Figure 1 shows the main segment of the Hilbert operator form and the resulting signal transformation [9, 10].



a **b**

Fig. 1. Hilbert transform: a) Hilbert operator; b) resultant and conjugate signals

In Figure 1, curve 1 represents the Hilbert operator $h(t)=\frac{1}{\pi t}$, curve 2 represents the original signal i(t), and curve 3 represents the conjugate signal [9]. Therefore, when the original signal *i(t)* passes through its maximum, $h[i(t)]$ the conjugate signal passes through zero.

A fault in the operation of an induction motor leads to periodic amplitude modulation of the current, which means that the disturbance-modulated signal can be represented as a cosine function. Thus, we describe the amplitude-modulated current waveform, where the amplitude is modulated according to the cosine rule:

$$i(t) = i_1 \cos(\omega_1 t)\left[1 + i_m \cos(\omega_m t)\right], \tag{1}$$

where i_1 – amplitude of the fundamental harmonic of the current, A; i_m – modulation depth, A; ω_m – cyclic modulation frequency, s^{-1}.

To extract the envelope of the current waveform, we use the Hilbert transform:

$$\overline{i_{oe}}(t) = i(t) + jh\left[i(t)\right], \tag{2}$$

where $h\left[i(t)\right]$ – imaginary component, which is the conjugate component to the output signal, which is determined by the direct Hilbert transform:

$$h\left[i(t)\right] = \frac{1}{\pi t} \cdot i(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{i(\tau)}{t-\tau} d\tau. \tag{3}$$

Equation (3) shows that the Hilbert transform is the result of convolution of the signal *i(t)* with the function $h(t) = \frac{1}{\pi t}$. Using (2) for the current waveform *i(t)*, the modulus and phase of its envelope will be calculated by the expressions:

$$A(t) = \sqrt{i^2(t) + h\left[i(t)\right]^2}, \tag{4}$$

$$\varphi(t) = arctg\left(\frac{h\left[i(t)\right]}{i(t)}\right). \tag{5}$$

Taking into account (4) and (5), equation (2) can be written as follows:

$$\overline{i_{oe}}(t) = A(t)e^{j\varphi(t)}. \tag{6}$$

Using (6), the analytical envelope signal for the current waveform (1) can be written as:

$$\overline{i_{oe}}(t) = i_1\left[1 + i_m \cos(\omega_m t)\right]e^{j(\omega_1 t)}. \tag{7}$$

The relation (7) shows that the analytic signal of the envelope contains periodic oscillations with modulation depth i_m and frequency ω_m, which characterize changes in the current waveform (1) as a result of the motor fault.

Since demodulation using the Hilbert transform allows for the examination of the spectrum without the main current component at the frequency f_1 [9, 10], taking into account the previous statements [6], we can write the analytical equations that characterize the changes in the currents due to periodic amplitude modulation.

For a faulty induction motor with bearing wear, we then have:

$$\begin{cases} i_a(t) = \sum_{\nu=1}^{\infty} i_{mmf}^{\nu} \cos(\omega_1 t - \phi_1)\left[\begin{array}{c} 1 + \sum_{\nu=1}^{\infty} i_{fr}^{\nu} \cos(\omega_r t - \phi_r) + \sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p}\right)t - \phi_{r\pm\lambda}\right) + \\ + \sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p} \pm \frac{kZ_r}{p}\right)t - \phi_{r\pm Zr}\right) \end{array}\right], \\ i_b(t) = \sum_{\nu=1}^{\infty} i_{mmf}^{\nu} \cos\left(\omega_1 t - \frac{2\pi}{3} - \phi_1\right)\left[\begin{array}{c} 1 + \sum_{\nu=1}^{\infty} i_{fr}^{\nu} \cos(\omega_r t - \phi_r) + \sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p}\right)t - \phi_{r\pm\lambda}\right) + \\ + \sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p} \pm \frac{kZ_r}{p}\right)t - \phi_{r\pm Zr}\right) \end{array}\right], \\ i_c(t) = \sum_{\nu=1}^{\infty} i_{mmf}^{\nu} \cos\left(\omega_1 t + \frac{2\pi}{3} - \phi_1\right)\left[\begin{array}{c} 1 + \sum_{\nu=1}^{\infty} i_{fr}^{\nu} \cos(\omega_r t - \phi_r) + \sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p}\right)t - \phi_{r\pm\lambda}\right) + \\ + \sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu} \cos\left(\omega_r\left(1 \pm \frac{k}{p} \pm \frac{kZ_r}{p}\right)t - \phi_{r\pm Zr}\right) \end{array}\right], \end{cases} \tag{8}$$

where the modulated oscillations of the component currents i_{fr}^{ν}, $i_{fr\pm\lambda}^{\nu}$ and $i_{fr\pm Zr}^{\nu}$ in a working induction motor approach zero.

Taking into account equation (7), we transform equation (8) to obtain the analytical relationships for the envelope waveforms of the currents under bearing wear:

$$\begin{cases} \overline{i_{a.oг}}(t)=\sum_{\nu=1}^{\infty} i_{mmf}^{\nu}\cos\left[\begin{array}{c} 1+\sum_{\nu=1}^{\infty} i_{fr}^{\nu}\cos(\omega_r t-\phi_r)+\sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\right)t-\phi_{r\pm\lambda}\right)+ \\ +\sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\pm\frac{kZ_r}{p}\right)t-\phi_{r\pm Zr}\right) \end{array}\right]\cdot e^{j(\omega_1 t-\phi 1)}, \\ \overline{i_{b.oг}}(t)=\sum_{\nu=1}^{\infty} i_{mmf}^{\nu}\cos\left[\begin{array}{c} 1+\sum_{\nu=1}^{\infty} i_{fr}^{\nu}\cos(\omega_r t-\phi_r)+\sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\right)t-\phi_{r\pm\lambda}\right)+ \\ +\sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\pm\frac{kZ_r}{p}\right)t-\phi_{r\pm Zr}\right) \end{array}\right]\cdot e^{j\left(\omega_1 t-\frac{2\pi}{3}-\phi 1\right)}, \\ \overline{i_{c.oг}}(t)=\sum_{\nu=1}^{\infty} i_{mmf}^{\nu}\cos\left[\begin{array}{c} 1+\sum_{\nu=1}^{\infty} i_{fr}^{\nu}\cos(\omega_r t-\phi_r)+\sum_{\nu=1}^{\infty} i_{fr\pm\lambda}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\right)t-\phi_{r\pm\lambda}\right)+ \\ +\sum_{\nu=1}^{\infty} i_{fr\pm Zr}^{\nu}\cos\left(\omega_r\left(1\pm\frac{k}{p}\pm\frac{kZ_r}{p}\right)t-\phi_{r\pm Zr}\right) \end{array}\right]\cdot e^{j\left(\omega_1 t+\frac{2\pi}{3}-\phi 1\right)}. \end{cases} \quad (9)$$

The obtained relations (9) show that the envelope waveforms of the currents contain a constant component of the resulting stator windings magnetomotive force and variable components of amplitude-modulated harmonics due to the motor fault.

When diagnosing the operation of an induction motor with worn bearings, several characteristic signs can be identified:

- harmonic components $f_1 \pm \nu \cdot f_r$, caused by current modulation and rotor rotation frequency;
- harmonic components $f_r\left(1\pm\frac{k}{p}\right)$, caused by periodic changes in the magnetic conductivity of the air gap due to eccentricity;
- harmonic components $f_r\left(1\pm\frac{k}{p}\pm\frac{kZ_r}{p}\right)$, are associated with the periodic interaction of the rotor tooth harmonics and changes in the magnetic conductivity of the air gap due to eccentricity.

Considering this, it is clear that in the expression $f_1 \pm \nu f_r$, the first component is the main harmonic of the supply network f_1, and the second component is modulated due to the fault. The Hilbert transform allows obtaining the envelope waveforms of currents and their spectra, which contain amplitude-modulated oscillations without the dominant component f_1.

When considering an induction motor with the number of pole pairs $2p=2$, the rotor speed is approximately equal to the supply network frequency $f_r \approx f_1$. Given this, in the spectrum of the envelope waveforms of the currents, the following amplitude-modulated oscillations of the current components should be analyzed when operating with bearing wear: $\nu \cdot f_r$, $\frac{\nu \cdot f_r}{p}$ і $f_r\left(\frac{k}{p}\pm\frac{kZ_r}{p}\right)$.

The study of the envelope waveform of the current allows for the analysis of the spectrum, which contains amplitude-modulated oscillations (low-frequency oscillations) in the absence of a dominant component at the supply network frequency f_1. It should be noted that amplitude-modulated oscillations, which characterize bearing wear, are also present in the classical spectrum based on the initial current waveform, but their comparative analysis is complicated due to the dominant current component.

For induction motors (IM) with healthy bearings, the amplitude-modulated oscillations at the frequencies $\nu \cdot f_r$, $\frac{\nu \cdot f_r}{p}$ і $f_r\left(\frac{k}{p}\pm\frac{kZ_r}{p}\right)$ are at the level of general noise within the spectral composition.

Therefore, based on the above, it can be concluded that the analysis of the envelope waveforms of the currents should be carried out with a bandwidth up to the maximum amplitude-modulated oscillations. Since only the amplitude-modulated oscillations are analyzed, without the main current component, the spectrum of the envelope waveform should be presented in a linear scale (A/Hz), which facilitates better clarity during graphical analysis, as opposed to the logarithmic scale (dB/Hz).

5. Conclusion

Diagnostic signs of bearing defects in induction motors have been proposed and substantiated, which can be used for real-time diagnostics during operation.

The theoretical justification for the application of the Hilbert transform in current-based diagnostics of bearing defects in induction motors has been provided. It has been shown that using this approach allows the extraction of the main component corresponding to the supply network frequency from the current waveform, leaving only the low-frequency harmonic components in the resulting envelope, which can be used as diagnostic signs for the investigated type of defects.

It has been demonstrated that the analysis of the envelope waveforms of the currents should be performed with a bandwidth up to the maximum amplitude-modulated oscillations, and the spectrum of the current envelope waveform should be presented in a linear scale (A/Hz), which enhances the clarity during their analysis.

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ЗАСТОСУВАННЯ ОГИНАЮЧИХ ДІАГРАМ СТАТОРНИХ СТРУМІВ В ЗАДАЧАХ ДІАГНОСТУВАННЯ ПІДШИПНИКІВ АСИНХРОННИХ ДВИГУНІВ

Сьогодні в умовах високих вимог до надійності та безперебійної роботи обладнання, електричні машини використовуються у важливих і навіть критичних технологічних процесах, де їх аварійна відмова може спричинити серйозні наслідки, включаючи зупинку виробництва, матеріальні збитки або навіть загрози для безпеки. Системи діагностики дають можливість своєчасно виявити можливі несправності, що дозволяє уникнути аварійних ситуацій та значно знизити витрати на непередбачувані ремонти.

Крім того, з розвитком технологій конструкція електричних машин стає все більш складною, що вимагає впровадження більш точних методів моніторингу. Така діагностика дозволяє не лише попередити виникнення аварійних ситуацій, а й подовжити термін служби обладнання. Крім цього, своєчасне виявлення дефектів допомагає підтримувати високу енергоефективність машин, що важливо для зниження витрат на енергоресурси та мінімізації негативного впливу на навколишнє середовище.

Інтеграція діагностичних систем у загальну автоматизовану інфраструктуру дозволяє проводити моніторинг у реальному часі та здійснювати більш точне прогнозування можливих відмов. Такі підходи знижують ризики аварій, покращують ефективність управління виробничими процесами та забезпечують безпеку. В умовах переходу до Індустрії 4.0 та широкого застосування цифрових технологій, такі системи стають незамінним елементом для забезпечення стабільної та безпечної роботи обертових електричних машин, що є важливим для сучасних промислових підприємств, а також дозволяють суттєво зменшити енергоємність виробничих процесів за рахунок продовження тривалості експлуатації основних виробничих фондів.

В роботі запропоновано діагностичні ознаки наявності дефектів підшипникових вузлів асинхронних двигунів, теоретично доведено доцільність застосування перетворення Гільберта при здійсненні струмової діагностики дефектів підшипників та показано, що спектр огинаючої осцилограми струму слід виконувати в лінійній шкалі.

Ключові слова: *діагностування, асинхронний двигун, дефект, підшипниковий вузол, аварійна відмова, огинаюча діаграми струму*.

Ф. 9. Рис. 1. Літ. 10.

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