№ 3 (134) / 2026
ISSN: 2520-6168 Vol. 134, № 3 / 2026
Техніка, енергетика, транспорт АПК
Technology, energy, agriculture transport AIC
UDC 631.319.4
DOI: 10.37128/2520-6168-2026-3-17

STUDY OF THE DYNAMIC CHARACTERISTICS OF A MULTIFUNCTIONAL ROTARY WORKING UNIT

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A mathematical model of the dynamics of a multifunctional rotary working unit was developed, and the influence of its structural and kinematic parameters on the critical shaft rotation speed and the amplitude-frequency characteristics was investigated. The working unit is considered as a complex mechanical system comprising an elastic shaft with discretely placed concentrated masses, as well as hinge-mounted flail hammers. The relevance of the study lies in establishing the characteristic rotation frequency zones under the condition of ensuring resonance-free operation of the rotor.

To construct the dynamic model, an energy-based approach and the Lagrange equations of the second kind were used. The model accounts for the energy of the shaft, the discretely placed masses, and the variable position of the flail hammers, as well as energy losses due to damping and the destruction of plant mass. As a result of the simulation, the equation of motion of the rotor was obtained, on the basis of which an expression was derived for determining the critical shaft rotation speed and the amplitude-frequency characteristic of the system.

A numerical experiment was conducted for an experimental rotor sample, which established that at an effective mass of 50.087 kg and a radial shaft stiffness of $2.426 \cdot 10^6$ *N/m, the critical rotation speed is 220.078 rad/s. The influences of effective mass, radial shaft stiffness, viscous damping coefficient, and imbalance magnitude on the position of the resonance zone and the oscillation amplitude were substantiated. It was established that an increase in the rotor's effective mass reduces the critical rotation speed, while an increase in shaft stiffness contributes to its growth. It is shown that an increase in damping significantly reduces the maximum oscillation amplitude in the resonance zone, whereas an increase in imbalance causes a proportional increase in vibrational loads.*

Based on the simulation results, it was established that the working range of the rotor rotation frequency of 1200-1300 rpm lies in the pre-resonance zone and is characterized by insignificant oscillation amplitudes. The obtained value of the safety factor with respect to the critical frequency is 1.617, which indicates a sufficient distance of the operating mode from the resonance region.

Keywords: *rotary working unit, milling-mulching drum, critical speed, rotor dynamics, shaft vibrations, imbalance, damping, mathematical model, experiment, process, plant residue shredding.*

Eq. 48. Fig. 6. Ref. 12*.*

1. Problem formulation

The implementation of technological processes in agricultural production is increasingly accompanied by the use of combined or multifunctional working units [12].

Considering the developed rotary working unit [3, 5], its multifunctionality is manifested in the fact that it can initially be used as a milling drum to perform conventional soil milling, and after minor re-equipment and a change in the kinematic operating mode, it can be used as a mulcher for plant residues.

Alongside such multifunctionality, the dynamic stability of the working unit to operate at high frequencies – which are necessary for achieving the specified interaction speeds of the flail elements with plant stems [2, 4] – changes drastically.

During operation, the rotary working unit constitutes a complex mechanical system that includes an elastic shaft with discretely arranged working elements and hinge-mounted flail hammers. The action of inertial forces, rotor imbalance, and impact interaction with plant mass causes shaft vibrations, which can bring

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Article received 29.07.26 Review completed 28.09.26 Article accepted 05.10.26 Article published 12.10.26

the system closer to resonance regimes and lead to a reduction in the reliability and durability of the structure. Therefore, the study of dynamic processes in such working units is an important scientific and practical task.

A pressing problem is the development of a mathematical model of the combined rotary working unit that accounts for the inertial characteristics of the rotor, the elasticity of the shaft, the damping properties of the system, and energy losses during the destruction of plant mass. Based on such a model, it is necessary to determine the critical rotation speed of the rotor and to assess the influence of the effective mass, radial shaft stiffness, imbalance magnitude, and damping on the amplitude-frequency characteristics of the system. Solving this problem makes it possible to substantiate safe operating modes of the rotary unit and to ensure a sufficient distance of the operating frequencies from the resonance zone.

2. Analysis of recent research and publications

A significant number of scientific works [7 – 10] have been devoted to the problems of rotor system dynamics, the determination of critical speeds, and the assessment of the influence of imbalance on shaft vibrations. The Jeffcott rotor model remains the basis for most modern studies, being used to analyze resonance phenomena, amplitude-frequency characteristics, and the rotor's passage through critical speeds. Contemporary works address issues of the stability of flexible rotors under variable angular velocity, as well as the influence of damping and bearing parameters on the dynamic behavior of the system. In particular, study [1] investigates the stability of Jeffcott rotor motion under variable rotation frequency and proposes performs a probabilistic analysis of critical speeds accounting for the uncertainty of the rotor's dynamic parameters.

At the same time, for agricultural rotary machines, the assessment of the influence of the structural parameters of working units on their dynamic characteristics acquires particular importance. Certain aspects of the functioning of mulching rotors are addressed in works [6], which investigate the aerodynamic processes in the working zone of the mulcher and the influence of the design of the working elements on the efficiency of the technological process. At the same time, the analysis of literary sources indicates that the issue of determining the critical speeds of combined milling-mulching drums, which have a complex structure with discretely placed masses and hinge-mounted flail hammers, has not been sufficiently covered. This necessitates the development of new mathematical models that account for the elasticity of the shaft, the inertial characteristics of the rotor, damping, and energy losses during the interaction of the working elements with plant mass.

3. The purpose of the article

The aim of this study is to develop a mathematical model of the dynamics of a multifunctional rotary working body and, based on this model, to investigate the influence of its design and kinematic parameters on the critical rotational speed of the shaft as well as on its amplitude-frequency characteristics.

4. Results and discussion

The combined rotary working unit represents a multicomponent mechanical system, the peculiarity of which lies in the fixed masses (disks, blades, and axles) discretely placed on a relatively long shaft, as well as movable masses in the form of flail hammers.

The objective of the study is to obtain the equation of motion of such a complex system, from which the critical rotation frequency of this rotor can be determined. Such a study is necessary in order to establish the correct kinematic operating mode of the working unit, avoiding resonance regimes.

To describe the system, we will use generalized coordinates: $q_1 = y(x,t)$ – transverse deflection of the shaft; $q_2 = \varphi(t)$ – angular rotation of the rotor; $q_3 = \alpha_i(t)$ – deflection angle of the i-th flail hammer;

Under these conditions, the kinetic energy of the system can be represented as the following relation [1]:

$$E = T + \Pi, \quad (1)$$

where T – kinetic energy of the system; Π – potential energy of the system.

The kinetic energy of the system is expressed as follows:

$$T = T_{sh} + T_{i\,m} + T_{hh}, \quad (2)$$

where T_{sh} – kinetic energy generated by the rotor shaft; $T_{i\,m}$ – kinetic energy of the masses discretely and fixedly positioned on the rotor shaft; T_{hh} – kinetic energy of the movable flail hammers.

The expressions for the energy components are presented below.

The kinetic energy of the shaft consists of two components – that due to shaft deflection T_d and that due to rotational motion T_r:

$$T_{sh} = T_d + T_r . \quad (3)$$

The first component of expression (3) characterizes the energy of shaft deflection $y(x,t)$, since the shaft is elastic; this can also be referred to as the energy of transverse vibrations of the shaft.

In this case, each elementary segment dx possesses a velocity $\dot{y}(x,t)$, and the mass of such an element is equal to $dm = \rho A dx$ consequently, the kinetic energy is given by the expression:

$$dT = \frac{1}{2} dm\dot{y}^2 = \frac{1}{2}\rho A\dot{y}^2 dx . \quad (4)$$

For the entire length of the shaft, the expression for the kinetic energy of shaft deflection is obtained as follows:

$$T_{sh} = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y}{\partial t}\right)^2 dx , \quad (5)$$

where L – length of the shaft; ρ – density of the shaft material; A – cross-sectional area of the shaft.

The kinetic energy due to rotational motion is:

$$T_r = \frac{1}{2} J_{sh}\dot{\varphi}^2 , \quad (6)$$

where J_{sh} – moment of inertia of the shaft; $\dot{\varphi}$ – angular velocity of the shaft.

In general, the kinetic energy of the shaft is described by the expression:

$$T_{sh} = \frac{1}{2}\left[\int_0^L \rho A\left(\frac{\partial y}{\partial t}\right)^2 dx + J_{sh}\dot{\varphi}^2\right]. \quad (7)$$

By analogy, the expression for the kinetic energy of the concentrated masses is derived as follows:

$$T_{i\,m} = \frac{1}{2} m_i\left(\dot{y}_i^2 + \omega^2 r_i^2\right), \quad (8)$$

where r_i – radius of the center of mass for the *i*- th mass, and angular velocity ω.

The kinetic energy of the hammers is characterized by the energy of rotation of the hammers together with the rotor under steady-state conditions, when they occupy a practically stationary radial position, and by the energy of the relative motion of the hammer about its mounting axis during deflection upon contact with plant mass.

Here we introduce the following notation: m_h – mass of the hammer; R_h – variable radius characterizing the position of the center of mass relative to the rotor's axis of rotation; $\vartheta_h = \omega R_h$ – linear velocity of the hammer's center of mass.

The kinetic energy of one hammer is then expressed as:

$$T_h = \frac{1}{2} m_h \omega^2 R_h^2 . \quad (9)$$

The expression R_h is accordingly given by:

$$R_h = R_0 + l\sin\alpha , \quad (10)$$

where R_0 – distance from the shaft's axis of rotation to the hammer's mounting axis; l – distance from the hammer's mounting axis to its center of mass; α – deflection angle of the hammer from the radial position.

The kinetic energy of the relative motion of the hammer is:

$$T_{rel} = \frac{1}{2} J_\alpha \dot{\alpha}^2 , \quad (11)$$

where J_α – moment of inertia of the hammer relative to the hinge; $\dot{\alpha}$ – angular velocity of hammer deflection relative to the hinge.

The total kinetic energy of the hammer is then given by:

$$T_{hh} = \frac{1}{2}\left(m_h \omega^2 R_h^2 + J_\alpha \dot{\alpha}^2\right). \quad (12)$$

We now present the expressions for the components of the potential energy of the system:

– potential energy due to shaft bending;

$$\Pi_{sh} = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx; \tag{13}$$

– potential energy of the hammers;

$$\Pi_h = m_h g l\left(1-\cos\alpha\right). \tag{14}$$

Air resistance and other dissipative forces are also of significant importance here. In general, they are expressed as follows:

$$D = \frac{1}{2}\left(c_p \dot{y}^2 + c_\alpha \dot{\alpha}^2\right) + D_t, \tag{15}$$

where c_p – damping coefficient of the rotor, accounting for friction in the bearings, aerodynamic resistance, and other resistances; c_α – damping coefficient of the hammer's hinge joint; D_t – energy losses due to the destruction of plant mass: stem cutting, shredding of plant residues, and impact deformation of the material.

The component is elaborated upon below D_t.

At the moment of contact between the hammer and the stem, part of its kinetic energy is lost. For example, per single impact:

$$\Delta E_t = \frac{1}{2} m_h\left(\vartheta_1^2 - \vartheta_2^2\right), \tag{16}$$

where ϑ_1 – velocity of the hammer before impact; ϑ_2 – velocity of the hammer after impact.

The corresponding power losses are then given by:

$$P_t = n_t \Delta E_t, \tag{17}$$

where n_t – number of impacts per unit time.

The obtained relation (17) is analogous to a dissipative function, and therefore it can be written as:

$$D_t = \frac{1}{2} n_t m_h\left(\vartheta_1^2 - \vartheta_2^2\right). \tag{18}$$

Considering that the impact restitution coefficient can be expressed as $k_v = \frac{\vartheta_2}{\vartheta_1}$, it follows that $\vartheta_2 = k_v \vartheta_1$. Expression (16) can then be rewritten as:

$$\Delta E_t = \frac{1}{2} m_h \vartheta_1^2\left(1-k_v^2\right). \tag{19}$$

On this basis, the energy loss expression (18) is written as:

$$D_t = \frac{1}{2} n_t m_h \vartheta_1^2\left(1-k_v^2\right). \tag{20}$$

Analyzing the physical meaning of the impact restitution coefficient, the following can be stated:

$k_v \to 1$ – the impact is elastic, and energy is almost not lost;

$k_v \to 0$ – the impact is perfectly inelastic, i.e., the energy is almost entirely converted into material destruction.

For interaction with plant mass, the value of this coefficient is $k_v = 0.3...0.6$.

Summarizing, the expression for the dissipative force (15) can finally be written in the form:

$$D = \frac{1}{2}\left[\left(c_p \dot{y}^2 + c_\alpha \dot{\alpha}^2\right) + n_t m_h \vartheta_1^2\left(1-k_v^2\right)\right]. \tag{21}$$

Attention is now returned to the principal problem, namely, the determination of the critical shaft speed.

The expression for the critical shaft speed is derived on the basis of the solution of the Lagrange equation of the second kind.

For the system under consideration, the Lagrange equation of the second kind is formulated as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} + \frac{\partial D}{\partial \dot{y}} = Q. \tag{22}$$

Here $L = T - \Pi$ – the Lagrangian.

In order to implement the solution of the Lagrange equation and determine the critical shaft speed, a number of assumptions are introduced: the shaft rotation frequency is such that the flail hammers can be assumed to occupy a radial position and to be rigidly fixed on their axes, i.e. $\alpha \approx 0$, $\dot{\alpha} \approx 0$. This assumption is valid since the centrifugal force:

$$F_c = m_h \omega^2 R_h >> m_h g . \tag{23}$$

Under these conditions, a single coordinate is sufficient for solving the problem of the critical shaft speed, namely $q = y(t)$, where $y(t)$ is the transverse deflection of the rotor axis.

The concept of the effective mass of the rotor is introduced, defined by the expression:

$$M_{ef} = \int_0^{L_v} \rho A \Phi^2(x) dx + \sum_{i=1}^{N} m_i \Phi^2(x_i), \tag{24}$$

where $\Phi(x)$ – the shape of the first bending mode.

The first term of expression (24) characterizes the mass of the shaft itself, while the second characterizes the mass of the remaining concentrated masses.

For convenience of presentation, an additional parameter is introduced, which characterizes the equivalent stiffness of the rotor. On the basis of beam bending theory, the expression for its potential energy, analogous to expression (13), is written, and, as in the case of the kinetic energy, a modal transformation is performed:

$$y(x,t) = \Phi(x) q(t), \tag{25}$$

then

$$\frac{d^2 y}{dx^2} = \frac{d^2 \Phi(x)}{dx^2} q(t), \tag{26}$$

The potential energy of the shaft then takes the form:

$$\Pi_{sh} = \frac{1}{2} q^2 \int_0^L EI \left(\frac{\partial^2 \Phi}{\partial x^2} \right)^2 dx, \tag{27}$$

where $K = \int_0^L EI \left(\frac{\partial^2 \Phi}{\partial x^2} \right)^2 dx$.

A compact expression for the potential energy of the shaft is thus obtained:

$$\Pi_{sh} = \frac{1}{2} K q^2 . \tag{28}$$

The Lagrangian of the system, in expanded form, is described by the expression:

$$L = \frac{1}{2} \left(J_\Sigma \omega^2 + M_{ef} \dot{y}^2 - K y^2 \right). \tag{29}$$

Next, the components of the Lagrange equation (22) are written as:

$$\begin{aligned} &\frac{\partial L}{\partial \dot{y}} = M_{ef} \dot{y} ; \\ &\frac{d}{dt} \left(M_{ef} \dot{y} \right) = M_{ef} \ddot{y} ; \\ &\frac{\partial L}{\partial y} = -K y ; \\ &\frac{\partial D}{\partial \dot{y}} = c \dot{y} . \end{aligned} \tag{30}$$

On this basis, the equation of motion of the rotor shaft is written as:

$$M_{ef} \ddot{y} + c \dot{y} + K y = Q(t). \tag{31}$$

In the presence of eccentricity, a centrifugal force arises, expressed as:

$$Q(t) = m_e e \omega^2 \cos(\omega t), \tag{32}$$

where m_e – imbalance mass; e – magnitude of the shaft eccentricity.

The equation of motion can then be rewritten as follows:

$$M_{ef}\ddot{y} + c\dot{y} + Ky = m_e e\omega^2 \cos(\omega t). \quad (33)$$

The solution of equation (33) is sought using classical approaches.

Analyzing the right-hand side of equation (33), it can be seen that the forcing force due to the imbalance varies according to a harmonic law:

$$Q(t) = Q_0 \cos(\omega t), \quad (34)$$

where $Q_0 = m_e e\omega^2$.

The solution can be sought in the form:

$$y = A\cos(\omega t - \varphi_f), \quad (35)$$

where A – amplitude of oscillations; φ_f – phase shift.

Here we obtain:

$$\dot{y} = -A\omega \sin(\omega t - \varphi_f);$$
$$\ddot{y} = -A\omega^2 \cos(\omega t - \varphi_f). \quad (36)$$

Substituting the obtained expressions into the equation of motion (33) yields:

$$-M_{ef} A\omega^2 \cos(\omega t - \varphi_f) - cA\omega \sin(\omega t - \varphi_f) + KA\cos(\omega t - \varphi_f) = m_e e\omega^2 \cos(\omega t). \quad (37)$$

Performing standard transformations, the expression for the amplitude-frequency characteristic is obtained:

$$A(\omega) = \frac{m_e e\omega^2}{\sqrt{\left(K - M_{ef}\omega^2\right)^2 + \left(c\omega\right)^2}}. \quad (38)$$

The resulting relation (38) characterizes the shaft deflection with increasing rotation speed.

In the absence of damping ($c = 0$), then:

$$A = \frac{m_e e\omega^2}{\left|K - M_{ef}\omega^2\right|}. \quad (39)$$

The theoretical condition for the amplitude to tend toward infinity is:

$$K - M_{ef}\omega^2 = 0, \quad (40)$$

from which the critical shaft speed accordingly follows as:

$$\omega_{cr} = \sqrt{\frac{K}{M_{ef}}}. \quad (41)$$

The obtained expression (41) represents the first critical angular velocity of the rotor.

For a more practical evaluation of the rotor's rotation speed, it is expedient to rewrite expression (41) as the critical rotation frequency of the rotor:

$$n_{cr} = \frac{30}{\pi}\sqrt{\frac{K}{M_{ef}}}. \quad (42)$$

The vibration frequency of the shaft can also be expressed in Hz:

$$f_{cr} = \frac{1}{2\pi}\sqrt{\frac{K}{M_{ef}}}. \quad (43)$$

When selecting the kinematic operating mode of the rotary working unit in mulcher mode, it is convenient to use a dimensionless form when determining the critical speed and the oscillation amplitude.

The natural frequency of oscillation of the rotor shaft is denoted by ω_n and a dimensionless parameter is introduced as follows:

$$r = \frac{\omega}{\omega_n}. \quad (44)$$

where ω – operating frequency of the rotor.

The expression for the shaft oscillation amplitude can then be represented in dimensionless form as:

$$A = \frac{m_e e}{M_{ef}} \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\varsigma r\right)^2}}, \tag{45}$$

where ς – damping coefficient:

$$\varsigma = \frac{c}{2\sqrt{K M_{ef}}}. \tag{46}$$

If the system exhibits weak damping and the rotor frequency coincides with the natural frequency of shaft oscillation, i.e. $r \approx 1$ and, accordingly, $\omega \approx \omega_n$, this represents one of the most hazardous operating modes of the rotor – the possibility of resonance occurring.

A practical calculation is performed, and the influence of changes in the rotor mass and its stiffness on the magnitude of the critical shaft speed is established.

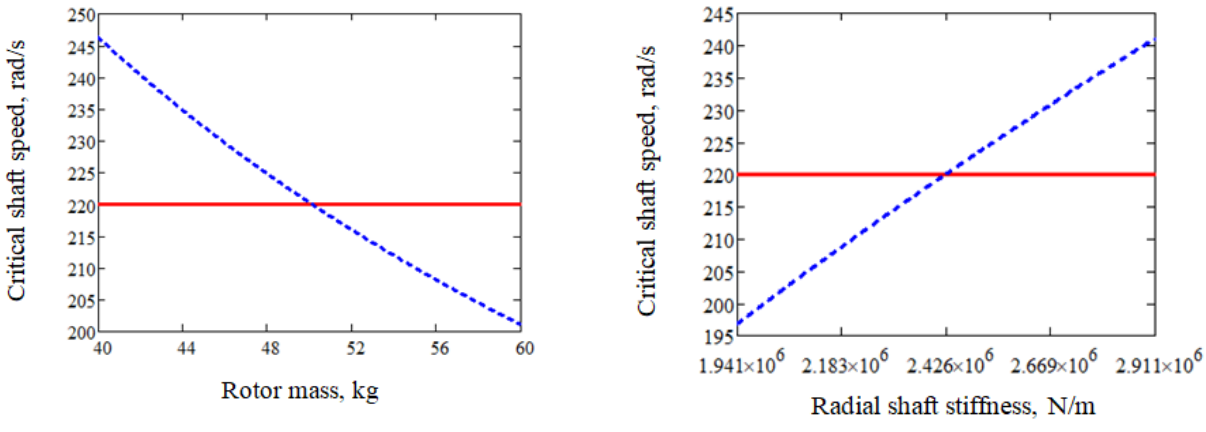
For the numerical experiment, the parameters of the experimental prototype of the rotary working unit were used: rotor shaft length of $L = 1.8$ m; outer diameter of the steel tubular shaft of $D = 0.09$ m, inner diameter of $d = 0.078$ m; shaft mass of $M_{sh} = 23.226$ kg; disk mass of $m_d = 2.806$ kg, quantity – 8 pcs.; blade mass of $m_n = 0.822$ kg, quantity – 32 pcs.; axle mass of $m_o = 0.663$ kg, 28 pcs.; flail hammer mass of $m_h = 0.301$ kg, quantity – 84 pcs.

On this basis, the effective mass of the rotor was found, amounting to:

$$M_{ef} = \frac{M_{sh}}{2} + \sum_{j=1}^{8} m_d \sin^2\left(\frac{\pi x_j}{L}\right) + \sum_{k=1}^{32} m_n \sin^2\left(\frac{\pi x_k}{L}\right) + \\ + \sum_{l=1}^{28} m_o \sin^2\left(\frac{\pi x_l}{L}\right) + \sum_{p=1}^{84} m_h \sin^2\left(\frac{\pi x_p}{L}\right). \tag{47}$$

Following the calculation, $M_{ef} = 50.087$ kg, and the radial shaft stiffness is $K = 2.426 \cdot 10^6$ N/m.

The influence of these parameters on the critical shaft speed is now shown, with their values tabulated within a range of ±20 %, Fig. 1.



a **b**

Fig.1. Study of the influence of shaft mass (a) and its radial stiffness (b) on the critical rotation speed

In Fig. 1, a, the solid line shows the value of the critical shaft speed ($\omega_{cr} = 220.1$ rad/s) found from relation (41) for the parameters presented above; the dashed line shows the change in the critical shaft speed value for the corresponding values of the rotor's effective mass and the radial stiffness of its shaft.

Analyzing the presented graphs, it can be seen that as the rotor mass decreases, the value of its critical rotation speed increases ($M_{ef} = 40.0$ kg – $\omega_{cr} = 246.3$ rad/s), and conversely, an increase in the rotor mass decreases the critical rotor rotation speed ($M_{ef} = 60.0$ kg – $\omega_{cr} = 201.1$ rad/s); the radial shaft stiffness exhibits an inverse property (Fig. 1, b) – a decrease in shaft stiffness reduces its critical speed ($K = 1.94 \cdot 10^6$ N/m – $\omega_{cr} = 196.9$ rad/s); and conversely, an increase in stiffness increases the critical rotor speed ($K = 2.91 \cdot 10^6$ N/m – $\omega_{cr} = 241.1$ rad/s).

For the analysis of the operation of the rotary working unit, it is important not only to determine the magnitude of the critical shaft rotation speed, but also to assess the influence of the structural parameters of the drum on its dynamic behavior. It is precisely through the analysis of the amplitude-frequency characteristic that answers to the posed questions can be obtained.

The next stage of the numerical experiment is carried out with the following initial data and assumptions: the imbalance mass amounts to 5% of the total rotor mass; the eccentricity value is 0.002 m.

The change in the shaft oscillation amplitude at the critical rotation speed and at various viscous damping coefficients is analyzed, Fig. 2.

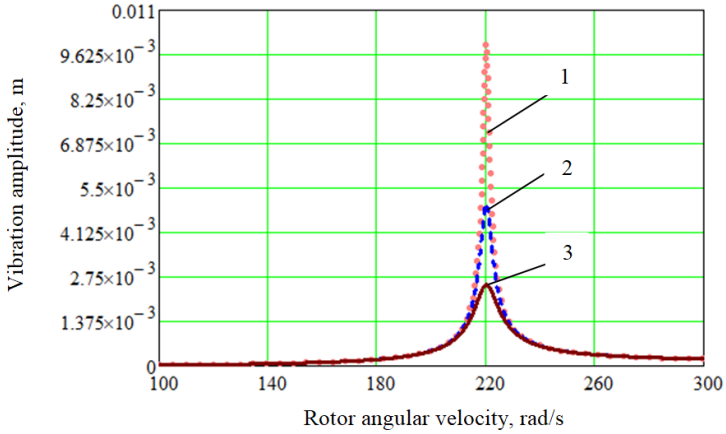


Fig. 2. Shaft oscillation amplitudes at different viscous damping coefficients

In Fig. 2, curve 1 shows the shaft oscillation amplitude at a viscous damping coefficient of $c = 110$ N·s/m, which corresponds to the mounting of the shaft on bearings attached to the frame. At the critical shaft rotation speed, the amplitude amounts to 0.01 m, and the resulting stresses exceed the permissible values by more than twofold. Even brief operation of the rotor under such conditions would lead to catastrophic failure.

For a viscous damping coefficient of $c = 220$ N·s/m (corresponding to a well-balanced shaft) under the same conditions, the amplitude amounts to 0.005 m (curve 2, Fig. 2). At such an amplitude (disregarding other factors), stresses close to the permissible limit would arise in the shaft. Under such conditions, short-term operation of the rotor may be permitted in exceptional cases.

And the third case – a viscous damping coefficient of $c = 440$ N·s/m (corresponding to a typical rotary machine) – yields an amplitude of 0.0025 m (curve 3, Fig. 2). The obtained amplitude value indicates that even at the critical shaft rotation speed, the machine retains its operability, provided that the effect of the sharp amplitude increase in this frequency zone is not amplified by other factors, such as an increase in imbalance due to soil buildup, etc.

Next, the influence of the rotor's effective mass on its oscillation amplitude is shown, Fig. 3.

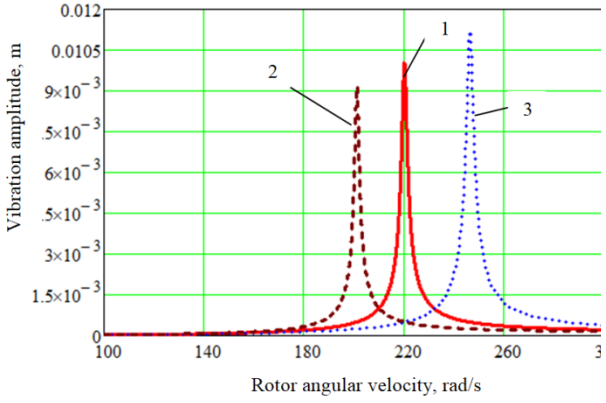


Fig. 3. Amplitude-frequency characteristic of the rotor shaft under the influence of changes in its effective mass

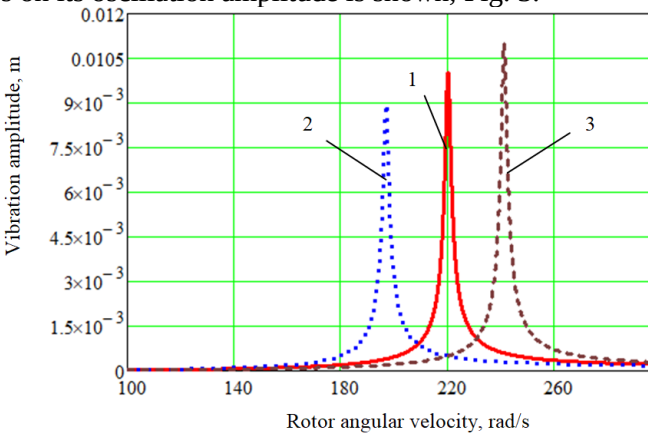


Fig. 4. Amplitude-frequency characteristic of the rotor shaft under the influence of changes in radial shaft stiffness

Figure 3 shows three curves characterizing the influence of the rotor's effective mass on its oscillation amplitude and on the location of the critical shaft speed zones: curve 1 is constructed for the actual effective mass of the rotor and a viscous damping coefficient of $c = 110$ N·s/m, for which the critical rotation speed is

220.087 rad/s and the maximum oscillation amplitude is 0.010 m; curve 2 is determined for the effective rotor mass increased by 20%: 201.085 rad/s, 0.009156 m; curve 3 is determined for the effective rotor mass decreased by 20%: 246.28 rad/s, 0.011 m.

It follows from this that an increase in the rotor's effective mass lowers the value of the critical shaft rotation speed while simultaneously reducing the oscillation amplitude, and conversely, a decrease in mass makes it possible to increase the critical shaft speed but also increases the oscillation amplitude.

A similar comparative analysis can be presented for changes in the radial shaft stiffness, Fig. 4.

As can be seen from Fig. 4, the radial shaft stiffness exhibits an inverse character of influence relative to the determination of the critical shaft rotation speed: a 20% reduction in stiffness ($K = 1.9408 \cdot 10^6$ N/m) lowers the critical rotation speed to 196.853 rad/s (curve 2, Fig. 4); a 20% increase ($K = 2.9112 \cdot 10^6$ N/m) raises it to 241.092 rad/s. The critical speed and amplitude data are compared relative to the corresponding values determined for the actual radial shaft stiffness ($K = 2.426 \cdot 10^6$ N/m, curve 1).

Equally important is the performance of a comparative analysis showing the dependence of the change in the rotor shaft oscillation amplitude on the magnitude of imbalance, Fig. 5.

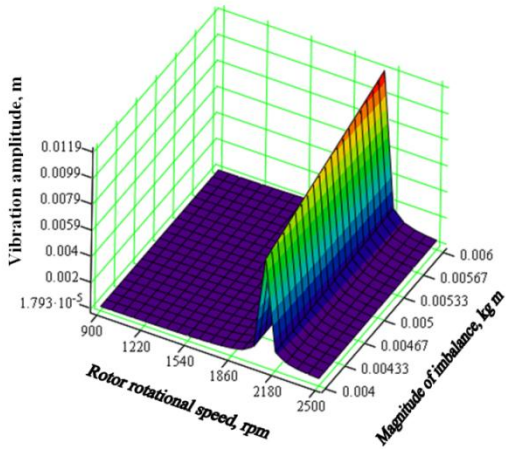


Fig. 5. Change in the rotor shaft oscillation amplitude with variation in the magnitude of imbalance

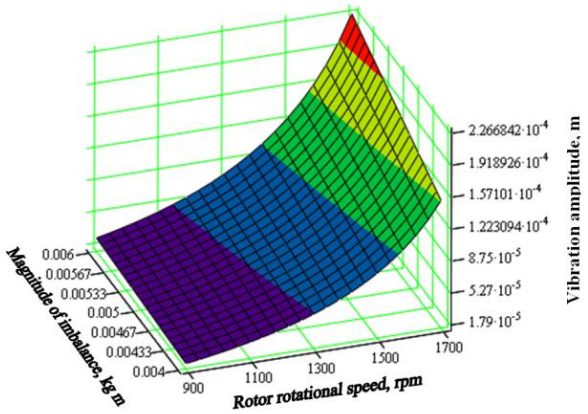


Fig. 6. Graphical dependence of the change in the rotor shaft oscillation amplitude within the range of operating frequencies

The result here is evident: a minimal imbalance value corresponds to a minimal oscillation amplitude. For example, at the critical rotor rotation frequency and a 20% reduced imbalance value (0.004 kg·m), the oscillation amplitude amounts to 0.008 m; at an increased imbalance value (0.006 kg·m), it amounts to 0.0012 m.

According to the preliminary calculations, the operating frequency of the rotor should lie within the range of 1200-1300 rpm. The calculated amplitude value within this frequency range amounts to $5.805 \cdot 10^{-5}$ m – $7.438 \cdot 10^{-5}$ m.

The critical rotor shaft rotation n_w frequency is then compared with the maximum frequency of the operating frequency range, yielding the value of the safety factor, which indicates the distance of the operating frequency from the resonance zone:

$$k_z \geq \frac{n_{cr}}{n_w} = \frac{2.102 \cdot 10^3}{1.3 \cdot 10^3} = 1.617 . \quad (48)$$

The value of the coefficient k_z is considered satisfactory if it lies within the range ≥ 1.25, i.e., for our rotor design and the substantiated kinematic operating mode, a significant distance of the operating frequency from the resonance zone is obtained.

It follows from this that the recommended rotor frequencies should lie within the pre-resonance zone. The theoretical limiting value of the operating shaft frequency is 2000 rpm, at which the oscillation amplitude amounts to 0.0011 m, taking into account the maximum calculated imbalance. However, an uncontrolled increase in rotor speed to 2100 rpm results in a sharp increase in amplitude to 0.012 m, which may already be destructive in nature. Post-resonance frequencies begin at 2200 rpm. This region was not investigated, since the linear velocity of the flail significantly exceeds the required velocity of impact interaction between the working edge and the plant stem, and increases the energy costs for driving the working unit.

5. Conclusion

In the conducted study, a theoretical model of the rotary working unit was constructed on the basis of energy-based approaches, accounting for energy losses due to the destruction of plant mass. The equation of motion of this rotor was formulated and solved, on the basis of which the critical rotation speed was determined to be 220.078 rad/s, or 2102 rpm. An analysis was performed of the influence of the rotor's viscous damping coefficient, effective mass, radial shaft stiffness, and imbalance magnitude on the value of the critical rotor rotation speed and on the amplitude of its oscillations. The numerical experiment shows that the pre-resonance zone frequency is ≤2000 rpm, and therefore the operating range of the rotor rotation frequency of 1200-1300 rpm lies within the pre-resonance zone and ensures stable operation of the working unit. The value of the safety factor with respect to the critical frequency is 1.617, which indicates a sufficient distance of the operating modes from the hazardous resonance region. For reliable operation of the rotary unit, it is recommended that its operating rotation frequency should not exceed 1700 rpm, which would correspond to a flail peripheral velocity of 57 m/s – a value satisfactory for shredding plant residues under the specified operating conditions of the machine.

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ДОСЛІДЖЕННЯ ДИНАМІЧНИХ ХАРАКТЕРИСТИК БАГАТОФУНКЦІОНАЛЬНОГО РОТАЦІЙНОГО РОБОЧОГО ОРГАНУ

У статті розроблено математичну модель динаміки багатофункціонального ротаційного робочого органу та досліджено вплив його конструктивно-кінематичних параметрів на величину критичної швидкості обертання вала та амплітудно-частотні характеристики. Робочий орган розглянуто як складну механічну систему, що містить пружний вал з дискретно розміщеними зосередженими масами, а також шарнірно закріпленими бильними молотками. Актуальність дослідження зумовлена тим, щоб встановити характерні зони частот обертання з умовою забезпечення безрезонансної роботи ротора.

Для побудови динамічної моделі використано енергетичний підхід та рівняння Лагранжа другого роду. У моделі враховано енергію вала, дискретно розміщених мас та змінне положення бильних молотків, а також втрати енергії на демпфування та руйнування рослинної маси. У результаті моделювання отримано закон руху ротора, на основі якого отримано вираз для визначення критичної швидкості обертання вала та амплітудно-частотної характеристики системи.

Проведено числовий експеримент для дослідного зразка ротора, де встановлено, що при його ефективній масі 50,087 кг, радіальній жорсткості вала $2{,}426 \cdot 10^6$ *Н/м – критична швидкість обертання становить 220,078 рад/с. Обґрунтовано впливи ефективної маси, радіальної жорсткості вала, коефіцієнта в'язкого демпфування та величини дисбалансу на положення резонансної зони та амплітуду коливань. Встановлено, що збільшення ефективної маси ротора знижує критичну швидкість обертання, а підвищення жорсткості вала сприяє її зростанню. Показано, що збільшення демпфування суттєво зменшує максимальну амплітуду коливань у зоні резонансу, тоді як зростання дисбалансу викликає пропорційне збільшення вібраційних навантажень.*

За результатами моделювання встановлено, що робочий діапазон частот обертання ротора 1200-1300 об/хв знаходиться в дорезонансній зоні та характеризується незначними амплітудами коливань. Отримане значення коефіцієнта запасу за критичною частотою становить 1,617, що свідчить про достатню віддаленість робочого режиму від резонансної області.

Ключові слова: *ротаційний робочий орган, фрезерно-мульчувальний барабан, критична швидкість, динаміка ротора, коливання вала, дисбаланс, демпфування, математична модель, експеримент, процес, подрібнення рослинних решток.*

Ф. 48. Рис. 6. Літ. 12.

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