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ANALYTICAL SUBSTANTIATION OF THE INFLUENCE OF PULSATION PARAMETERS AND TEAT LINER CHARACTERISTICS ON MILK FLOW INTENSITY

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The article presents the results of theoretical studies of the milk flow process in the biotechnical system "animal – milking machine". An analytical substantiation of the influence of pulsation parameters and the elastic-deformation characteristics of the teat liner on milk flow intensity was carried out. The machine milking process is considered as a complex dynamic system in which the operating vacuum, pulsation modes, teat liner tension force, and physiological characteristics of the animal teat interact simultaneously. Bernoulli's equation and hydrodynamic analysis methods were used to describe milk movement through the teat canal. The variation in milk flow rate during the pulsation cycle was described using a piecewise-continuous function that takes into account the phases of flow increase, stabilization, decrease, and residual milk leakage. The elastic-deformation state of the teat liner was determined on the basis of Hooke's law and the equations of deformable body mechanics. Analytical dependences were obtained that make it possible to evaluate the influence of the operating vacuum, pulsation phase duration, and teat liner tension force on milk flow velocity. Based on the results of theoretical calculations, rational operating parameters of the milking equipment were determined, ensuring maximum milk flow intensity while minimizing mechanical stress on teat tissues. The obtained results can be used for improving the design of milking machines and increasing the efficiency of machine milking.

The proposed theoretical model makes it possible to comprehensively evaluate the relationship between vacuum режим parameters, pulsation characteristics, and the mechanical effect of the teat liner on teat tissues. The research results can be used to substantiate energy-efficient and physiologically safe operating modes of milking machines in modern machine milking systems.

***Keywords**: machine milking, milk flow, milking machine, pulsation parameters, teat liner, operating vacuum, mathematical modeling, hydrodynamics, elastic-deformation state, teat sphincter.*

Eq. 14. Fig. 3. Ref. 17.

1. Problem formulation

The efficiency of machine milking is largely determined by the coordinated functioning of the elements of the biotechnical system "animal – milking machine". During machine milking, the teat is simultaneously affected by vacuum pressure, pulsation parameters, elastic-deformation characteristics of the teat liner, vacuum fluctuations in the pulsation chamber, and the physiological condition of the mammary gland. Therefore, the milk flow process should be considered as a complex dynamic system in which a change in one parameter causes variations in milk flow rate, milking duration, and the level of mechanical stress on teat tissues [1].

During the operation of the milking machine, the vacuum ensures the opening of the teat sphincter and the transportation of milk through the teat canal. At the same time, the pulsator creates a cyclic alternation of suction and compression phases, which provides periodic unloading of teat tissues. Insufficient vacuum stability, improper pulsation ratio, or excessive compression force of the teat liner may lead to congestion in teat tissues, hyperkeratosis, sphincter injury, and an increased risk of mastitis. Scientific studies indicate that high vacuum levels in the teat canal area and significant pressure fluctuations increase mechanical stress on tissues and worsen udder condition [2].

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It has been established that the main operating parameters determining the efficiency of the machine milking process are the level of operating vacuum, pulsation frequency, and the ratio of suction and compression phases. An increase in vacuum pressure enhances milk flow intensity; however, excessive vacuum causes overload of teat tissues and may damage the teat canal. Similarly, irrational pulsation parameters can result in incomplete closing or opening of the teat liner, thereby disturbing the physiological milking regime. Effective pulsation should ensure an optimal alternation between milk extraction phases and teat rest periods [3].

Particular attention should be paid to the selection of teat liner design parameters and liner tension force. The characteristics of the teat liner determine the stability of the vacuum beneath the teat, the magnitude of compressive force, and the intensity of mechanical action on udder tissues. Studies show that liner stiffness, teat cup geometry, and pulsation parameters significantly affect milk flow, teat canal condition, and the risk of mammary gland infection [4, 5].

In addition, vacuum instability and teat cup liner slips may cause reverse milk flow and the penetration of bacterially contaminated particles into the teat canal. This is considered one of the main causes of mastitis development when milking system parameters are improperly adjusted [6, 7].

Thus, improving the efficiency of machine milking requires a comprehensive consideration of the relationship between operating vacuum, pulsation parameters, teat liner design characteristics, and the physiological features of the mammary gland. In this regard, the development of a theoretical model of the milk flow process is highly relevant, as it would allow the determination of rational operating modes of milking equipment and ensure maximum milking intensity with minimal injury to teat tissues.

2. Analysis of recent research and publications

Theoretical studies of the milk flow process were carried out using methods of mathematical modeling, hydrodynamics, elasticity theory, and deformable body mechanics. Such an integrated approach made it possible to consider the machine milking process as a multifactor dynamic system in which the biological characteristics of the animal's mammary gland and the technical parameters of milking equipment interact simultaneously. The research was based on the development of an analytical model of milk movement through the teat canal, taking into account the influence of vacuum pressure, pulsation parameters, and the elastic-deformation characteristics of the teat liner [8, 9].

During the analysis of milk movement through the teat canal, Bernoulli's equation was used, which makes it possible to describe the relationship between milk flow velocity, pressure differential, and the geometric parameters of the teat sphincter. The application of a hydrodynamic model allowed the determination of stable milk flow conditions and the evaluation of the influence of operating vacuum on milk flow intensity. A similar approach is widely used in modern studies of machine milking processes and flow hydrodynamics in biotechnical systems [10, 11].

To describe changes in milk flow rate during the pulsation cycle, a piecewise-continuous function was used, taking into account the phases of flow increase, stabilization, decrease, and residual milk leakage. Such a mathematical approximation makes it possible to adequately describe real milk extraction processes and consider the nonuniform passage of milk through the teat sphincter during different phases of the suction cycle. To determine the total volume of milk extracted during one pulsation cycle, the flow rate function was integrated over time. Similar mathematical description methods are used in modeling pulsation processes and vacuum fluctuations in milking systems [12, 13].

The elastic-deformation state of the teat liner was determined based on the equations of deformable body mechanics and Hooke's law. In this case, the Young's modulus of the liner material, Poisson's ratio, geometric parameters of the teat liner, and liner tension force were taken into account. This approach made it possible to evaluate the influence of teat liner deformation on changes in the internal diameter of the teat sphincter and the magnitude of mechanical stress on teat tissues. It is known that liner stiffness and compression parameters significantly affect teat canal condition, vacuum stability, and the risk of mastitis occurrence [14, 15].

Based on the obtained analytical dependences, theoretical calculations were performed to determine the influence of operating vacuum, pulsation frequency, pulsation ratio, and teat liner tension force on milk flow intensity. Methods of mathematical analysis, numerical integration, and computer simulation were used to determine rational operating modes of the milking equipment. Graphical dependences were constructed using parametric analysis of functions, which made it possible to establish optimal variation ranges for the main operating parameters of the milking system. Similar modeling methods are actively used in modern studies of automatic milking systems and the interaction processes between teat liners and teats [16, 17].

The use of mathematical modeling and hydrodynamic approaches made it possible to obtain a generalized theoretical model of the milk flow process that considers the relationship between vacuum parameters, pulsation modes, teat liner characteristics, and milk extraction rate. This provides a basis for further improvement of milking machine designs and enhancement of machine milking efficiency.

3. The purpose of the article

To theoretically substantiate the influence of pulsation parameters, operating vacuum, and teat liner tension force on milk flow intensity and to determine rational operating modes of the milking system.

4. Results and discussion

Theoretical studies of the milk flow process were carried out using methods of mathematical modeling, hydrodynamics, elasticity theory, and deformable body mechanics. Such an integrated approach made it possible to consider the machine milking process as a complex biotechnical system in which the physiological characteristics of the animal’s mammary gland, the parameters of the vacuum regime, and the design features of milking equipment interact simultaneously. The research was based on the development of an analytical model of the milk extraction process through the teat canal, taking into account the influence of operating vacuum, pulsation parameters, and the elastic-deformation properties of the teat liner. To simplify the mathematical description of the process, it was assumed that milk is an incompressible fluid with constant density and that the flow in the teat canal has a laminar character.

During the analysis of milk movement through the teat canal, Bernoulli’s equation was used, which makes it possible to establish the relationship between milk flow velocity, pressure differential, and the geometric parameters of the teat sphincter. The application of the hydrodynamic approach made it possible to determine the conditions for stable milk flow and to evaluate the influence of vacuum pressure on milk flow intensity. Based on the obtained dependences, the velocity of the milk flow was determined depending on the teat canal diameter, vacuum level, and the pressure differential between the udder cistern and the teat chamber. In addition, the influence of the geometric parameters of the teat on the nature of milk passage through the sphincter was taken into account.

The mathematical description of milk flow rate variation during the suction phase was performed using a piecewise-continuous function that considers the phases of flow increase, stabilization, decrease, and residual milk leakage. Such an approximation made it possible to adequately describe the real milk extraction processes during one pulsation cycle and to account for the nonuniform passage of milk through the teat sphincter at different moments in time. To determine the total volume of milk extracted during one pulsation cycle, the flow rate function was integrated over the time interval of the pulsation cycle. This made it possible to establish the dependence of the extracted milk quantity on the duration of the suction and compression phases, as well as on the pulsation frequency of the milking machine.

The elastic-deformation state of the teat liner was determined using the equations of deformable body mechanics and Hooke’s law. In this case, the Young’s modulus of the liner material, Poisson’s ratio, teat liner tension force, and its geometric parameters were taken into account. The performed analysis made it possible to determine changes in the internal diameter of the teat sphincter under the action of compressive forces generated by the teat liner. The obtained dependences allowed the evaluation of the level of mechanical stress on teat tissues and the determination of conditions under which stable sphincter opening is ensured without excessive injury to mammary gland tissues.

Based on the obtained analytical dependences, theoretical calculations were performed to evaluate the influence of operating vacuum, pulsation frequency, pulsation ratio, and teat liner tension force on milk flow intensity. Methods of mathematical analysis, numerical integration, and computer simulation were used to determine rational operating parameters of the milking system. Graphical dependences were constructed through parametric analysis of functions, which made it possible to establish the optimal ranges of variation for the main operating parameters of the milking system. In particular, the influence of operating vacuum, pulsation phase duration, and teat liner tension force on milk flow intensity and the stability of the functioning of the biotechnical system “animal – milking machine” was determined.

The use of mathematical modeling methods and hydrodynamic analysis made it possible to obtain a generalized theoretical model of the milk flow process that considers the relationship between vacuum regime parameters, pulsation system characteristics, elastic-deformation properties of the teat liner, and milk

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extraction rate. The obtained results provide a theoretical basis for further improvement of milking machine designs, optimization of machine milking regimes, and enhancement of the efficiency of milking equipment operation.

The efficiency of machine milking is largely determined by the coordinated operation of the biotechnical system "animal – milking machine". During milking, the mammary gland is simultaneously affected by vacuum pressure, pulsation parameters, elastic-deformation characteristics of the teat liner, and the physiological characteristics of the animal. Therefore, the milk flow process should be considered as a complex dynamic system in which a change in one parameter causes changes in milking rate, milk flow intensity, and the level of stress on teat tissues.

During the operation of the milking machine, an alternating vacuum is created in the teat chamber, which ensures periodic opening of the teat sphincter and transportation of milk through the teat canal. At the same time, the nature of milk flow depends not only on the vacuum level, but also on the pulsation frequency, the ratio of suction and compression phases, the stiffness of the teat liner, and the magnitude of its tension force.

Analysis of the functioning process of the milking system indicates that the intensity of milk extraction from the cow's udder under the action of the milking machine can be described by the following functional relationship:

$$V = q \cdot n \cdot N_c, \tag{1}$$

V – milk flow intensity, m³/s; q – volume of milk passing through the sphincter during one pulsation cycle, m³/cycle; n – pulsation frequency of the milking machine, cycles/s; N_c – number of teats.

After determining the components of the right-hand side of equation (2.1), it is possible to obtain an analytical dependence of milk flow intensity on the design parameters of the teat liner and the operating modes of the milking unit. Investigation of this dependence makes it possible to evaluate the degree of influence of the operating vacuum, pulsation frequency, pulsation ratio, and teat liner tension force on the efficiency of the milking process and the stability of milk extraction.

The theoretical analysis of the process was carried out under the following assumptions:

– the pressure in the udder cistern during the active milk flow period remains constant;

– the teat cistern and the opened sphincter are considered as cylindrical channels;

– milk is regarded as an incompressible fluid with constant density;

– oscillations of the teat liner during pulsation are periodic and do not cause turbulent flow conditions.

During the study of milk passage through the teat canal in the suction phase (Fig. 1), the following notations were adopted: P_0 – milk pressure in the udder cistern, Pa; P – pressure in the teat chamber, Pa; d – diameter of the teat canal, m; v – milk flow velocity through the sphincter, m/s; L – teat length, m; Z_0, Z – hydraulic heads, m.

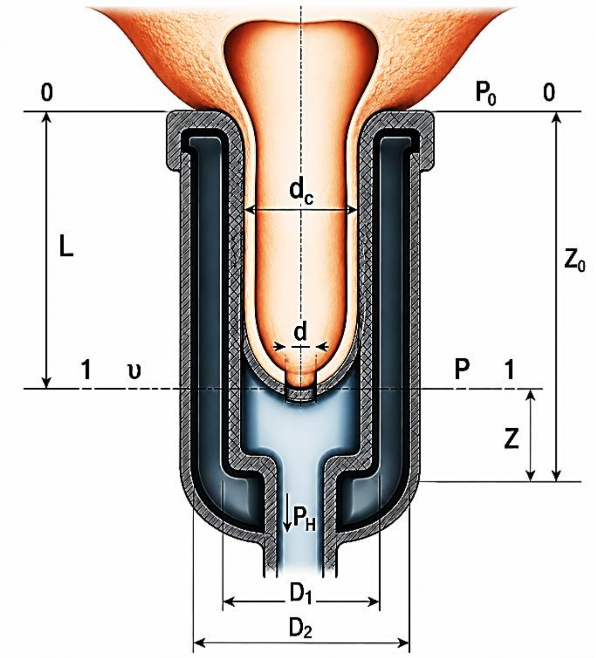

Fig. 1. Diagram of the interaction between the teat and the teat liner

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To determine the parameters of the milk flow, Bernoulli's equation was applied to section 0–0, which passes along the teat canal.

$$Z_0 + \frac{P_0}{\rho g} = Z + \frac{P}{\rho g} + \frac{v^2}{2g}, \tag{2}$$

where ρ is the milk density, kg/m³; g is the gravitational acceleration, m/s².

Let us transform equation (2) into the following form

$$\frac{P_0 - P}{\rho g} + L = \frac{\upsilon^2}{g}, \tag{3}$$

where $L = Z_0 - Z$.

The milk flow velocity can be expressed through the maximum milk flow rate Q_{max} and the diameter of the teat canal

$$\upsilon = \frac{4Q_{max}}{\pi d^2}. \tag{4}$$

Substituting equation (4) into expression (3) and performing the corresponding mathematical transformations, we obtain the dependence of the maximum milk flow rate on the pressure differential and the geometric parameters of the sphincter

$$Q_{max} = \frac{\pi d^2}{4}\sqrt{\frac{P_0 - P + \rho g L}{\rho}}. \tag{5}$$

It is assumed that the milk flow rate Q through the sphincter during one suction phase varies over time according to the Heaviside function (Fig. 2):

$$Q(t) = \begin{cases} Q_{max}\left(1 - e^{-k_1 t}\right), & 0 \le t < t_a, \\ Q_{max}, & t_a \le t < t_a + t_b, \\ Q_{max} e^{-k_2 (t - t_b)}, & t_a + t_b \le t < t_a + t_b + t_c, \\ Q_{max}\alpha(t - t_c)^2, & t_a + t_b + t_c \le t < t_a + t_b + t_c + t_d. \end{cases} \tag{6}$$

where k_1 is the growth coefficient, s^{-1}; k_2 is the decay coefficient, s^{-1}; α is the curvature parameter of the final section, s^{-1}; t_a, t_b, t_c, t_d are the time boundaries of the phases, s.

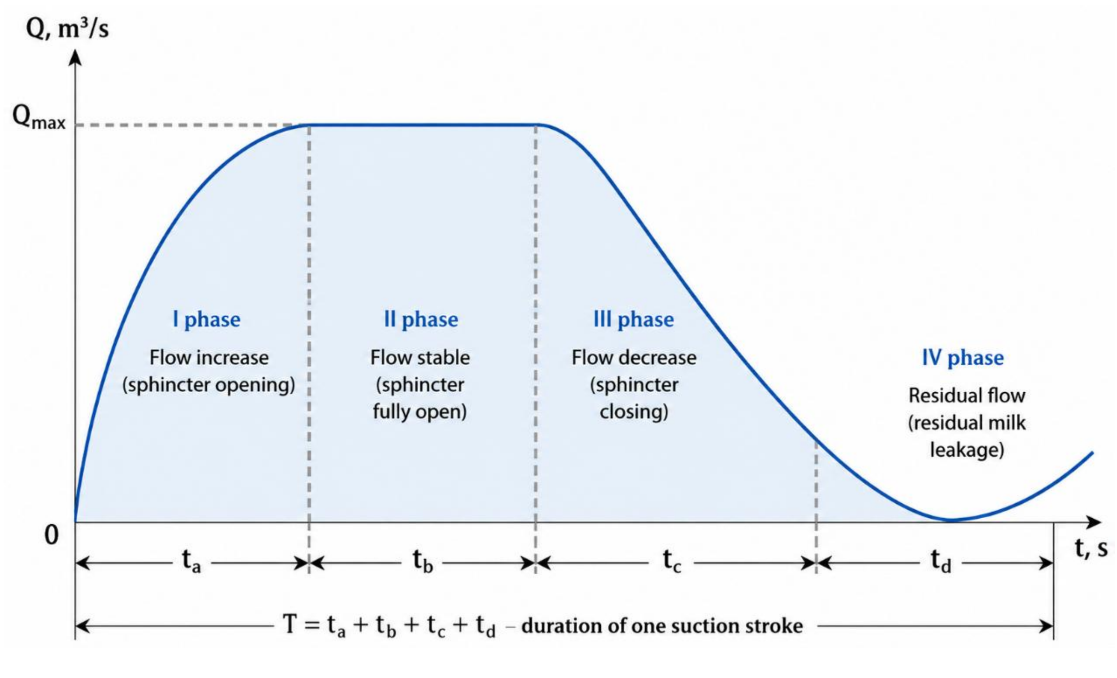

Fig. 2. Graph of milk flow rate variation through the teat sphincter during the suction phase

To determine the total volume of milk released during one cycle, expression (6) was integrated over time

$$\begin{aligned} q &= \int_0^{t_a} Q_{max}(1-e^{-k_1 t})dt + \int_{t_a}^{t_a+t_b} Q_{max}dt + \int_{t_a+t_b}^{t_a+t_b+t_c} Q_{max}e^{-k_2(t-t_b)}dt + \int_{t_a+t_b+t_c}^{t_a+t_b+t_c+t_d} Q_{max}\alpha(t-t_c)^2 dt = \\ &= Q_{max}\left[t_a + \frac{e^{-k_1 t_a}-1}{k_1}\right] + Q_{max}t_b + \frac{Q_{max}}{k_2}\left[e^{-k_2 t_a} - e^{-k_2(t_a+t_c)}\right] + \frac{Q_{max}\alpha}{3}\left[(t_a+t_b+t_d)^3 - (t_a+t_b)^3\right] = \\ &= Q_{max}\left[\left[t_a + \frac{e^{-k_1 t_a}-1}{k_1} + t_b\right] + \frac{1}{k_2}\left(e^{-k_2 t_a} - e^{-k_2(t_a+t_c)}\right) + \frac{\alpha}{3}\left[(t_a+t_b+t_d)^3 - (t_a+t_b)^3\right]\right]. \end{aligned} \quad (7)$$

Substituting (5) into (7), we obtain

$$q = \frac{\pi d^2}{4}\sqrt{\frac{P_0 - P + \rho g L}{\rho}}\left[\left[t_a + \frac{e^{-k_1 t_a}-1}{k_1} + t_b\right] + \frac{1}{k_2}\left(e^{-k_2 t_a} - e^{-k_2(t_a+t_c)}\right) + \frac{\alpha}{3}\left[(t_a+t_b+t_d)^3 - (t_a+t_b)^3\right]\right]. \quad (8)$$

To determine the internal diameter of the sphincter, it is necessary to take into account the action of the teat liner tension force on the teat surface. The pressure created by the deformation of the teat liner is determined by the following relationship

$$P_c = \frac{E_r \Delta D\left(D_2 - \Delta D - D_1\right)}{\left(D_2 + D_1\right)D_1} \quad (9)$$

where ΔD is the absolute deformation of the teat liner, m; E_r is the Young's modulus of the liner material, Pa; D_2, D_1 are the outer and inner diameters of the teat liner, respectively, m.

From the condition of mechanical equilibrium for the teat, we obtain

$$P_c = \frac{Pd - E_c \dfrac{d}{d_c}\left(d_c - d\right)}{d_c} \quad (10)$$

where d_c is the teat diameter, m; E_c is the elasticity modulus of teat tissues, Pa.

The absolute deformation of the teat liner can be represented as

$$\Delta D = \varepsilon \cdot D_2 = \mu \frac{F_N D_2}{E_r \dfrac{\pi}{4}\left(D_2^2 - D_1^2\right)} \quad (11)$$

where ε is the relative deformation; μ is Poisson's ratio; F_N is the teat liner tension force, N; S is the cross-sectional area of the liner.

Taking into account equations (9)–(11), we obtain an expression for determining the internal diameter of the sphincter

$$d = \frac{\left(E_c - P\right)d_c}{2E_c} - d_c\sqrt{\frac{\left(P - E_c\right)^2}{4E_c^2} + \frac{4\mu F_N D_2}{\pi D_1 E_c\left(D_2 + D_1\right)\left(D_2^2 - D_1^2\right)}\left(D_2 - D_1 - \frac{4\mu F_N D_2}{\pi E_\Gamma\left(D_2^2 - D_1^2\right)}\right)}. \quad (12)$$

Substituting the obtained dependences into the basic equation of milk flow intensity (8), we finally obtain the generalized model of the milking process

$$\begin{aligned} q = {} & \frac{\pi}{4}\sqrt{\frac{P_0 - P + \rho g L}{\rho}}\left[\left[t_a + \frac{e^{-k_1 t_a}-1}{k_1} + t_b\right] + \frac{1}{k_2}\left(e^{-k_2 t_a} - e^{-k_2(t_a+t_c)}\right) + \frac{\alpha}{3}\left[(t_a+t_b+t_d)^3 - (t_a+t_b)^3\right]\right] \times \\ & \times \left[\frac{\left(E_c - P\right)d_c}{2E_c} - d_c\sqrt{\frac{\left(P - E_c\right)^2}{4E_c^2} + \frac{4\mu F_N D_2}{\pi D_1 E_c\left(D_2 + D_1\right)\left(D_2^2 - D_1^2\right)}\left(D_2 - D_1 - \frac{4\mu F_N D_2}{\pi E_\Gamma\left(D_2^2 - D_1^2\right)}\right)}\right]^2. \end{aligned} \quad (13)$$

Thus, milk flow intensity is determined by the complex interaction of operating vacuum, pulsation frequency, pulsation ratio, teat liner tension force, and the geometric parameters of the teat canal. An increase in vacuum pressure contributes to an increase in milk flow velocity; however, excessive vacuum may cause injury to teat tissues. Similarly, increasing the pulsation frequency improves milk extraction intensity only up to a certain limit, after which the efficiency of the process decreases due to incomplete opening of the sphincter.

The pulsation frequency n (min^{-1}) and pulsation ratio δ can be expressed through the phase durations:

$$n = \frac{60}{t_a + t_b + t_c + t_d}, \qquad \delta = \frac{t_a + t_b}{t_c + t_d}. \tag{14}$$

Taking into account equation (14), as well as the results of substituting experimentally determined coefficients and design parameters of the milking system into equation (13), a rational range of operating modes of the milking equipment was established, ensuring maximum milk flow velocity: operating vacuum P=48–49 kPa, pulsation phases t_a = 106–111 ms, t_b = 435–446 ms, t_c = 211–224 ms, t_d = 287–296 ms, and teat liner tension force F_H=59.3–60.4 N. The theoretical curves showing the influence of these parameters on milk flow velocity are presented in Fig. 3.

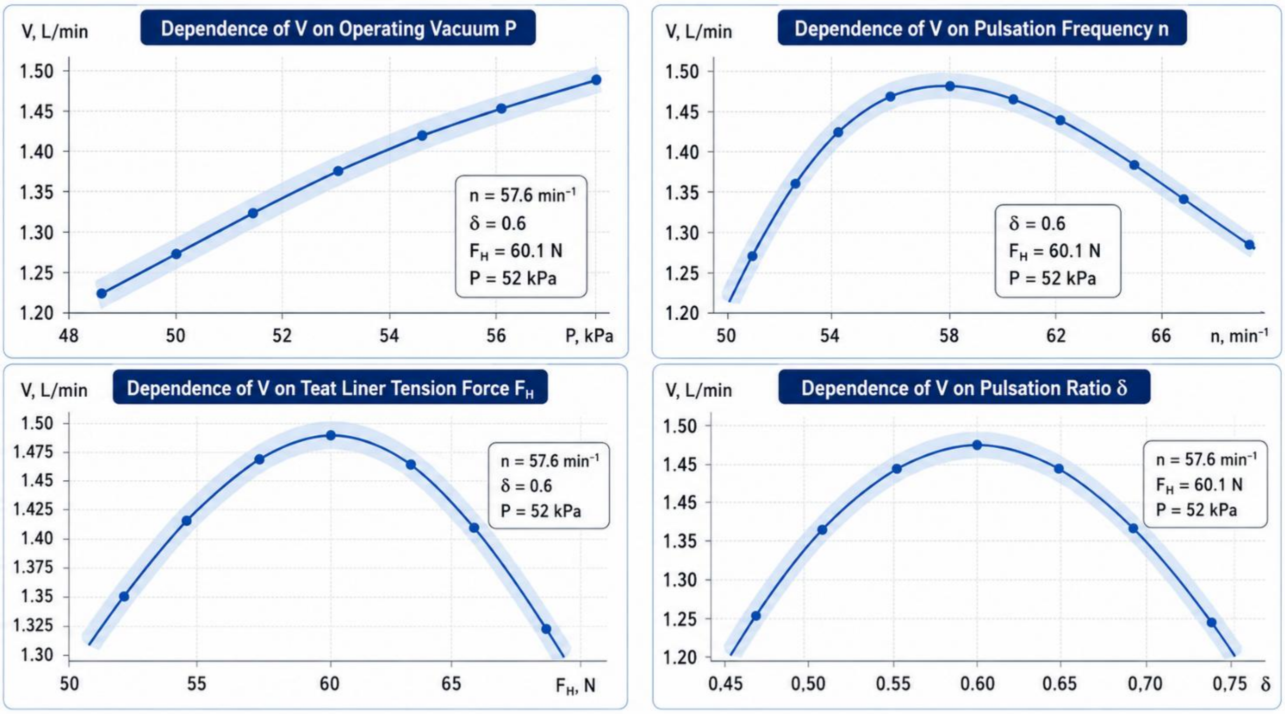

Fig. 3. Dependence of milk extraction velocity on the operating and design parameters of the milking system

5. Conclusion

As a result of the conducted theoretical studies, a mathematical model of the milk flow process in the biotechnical system "animal – milking machine" was developed, taking into account the influence of operating vacuum, pulsation parameters, and teat liner tension force on milk extraction intensity. It was established that the milk flow rate is determined by the complex interaction of the operating and design parameters of the milking system and depends on the dynamics of teat sphincter opening. Based on the application of Bernoulli's equation and the relationships of elasticity theory, analytical expressions were obtained describing milk movement through the teat canal and changes in the internal diameter of the sphincter under the action of the teat liner.

It was theoretically proven that an increase in operating vacuum contributes to an increase in milk extraction rate; however, excessive growth of vacuum pressure causes additional stress on teat tissues and may negatively affect the physiological condition of the udder. It was determined that a rational ratio of pulsation phases ensures a stable mode of milk passage through the teat canal and increases the efficiency of the milking process.

The obtained results can be used for improving the design of milking machines, optimizing machine milking modes, and increasing the operational efficiency of milking equipment.

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АНАЛІТИЧНЕ ОБҐРУНТУВАННЯ ВПЛИВУ ПАРАМЕТРІВ ПУЛЬСАЦІЇ ТА ДІЙКОВОЇ ГУМИ НА ІНТЕНСИВНІСТЬ МОЛОКОВІДДАЧІ

У статті наведено результати теоретичних досліджень процесу молоковіддачі в біотехнічній системі «тварина – доїльний апарат». Виконано аналітичне обґрунтування впливу параметрів пульсації та пружно-деформаційних характеристик дійкової гуми на інтенсивність виведення молока. Процес машинного доїння розглянуто як складну динамічну систему, у якій взаємодіють робочий вакуум, режими пульсації, сила натягу дійкової гуми та фізіологічні особливості дійки тварини. Для опису руху молока через дійковий канал використано рівняння Бернуллі та методи гідродинамічного аналізу. Зміну витрати молока протягом циклу пульсації описано за допомогою кусочно-неперервної функції, що враховує фази наростання, стабілізації, спаду та залишкового витікання молока. Пружно-деформаційний стан дійкової гуми визначали на основі закону Гука та рівнянь механіки деформівного тіла. Отримано аналітичні залежності, які дозволяють оцінити вплив робочого вакууму, тривалості фаз пульсації та сили натягу дійкової гуми на швидкість молоковіддачі. За результатами теоретичних розрахунків визначено раціональні параметри роботи доїльного обладнання, за яких забезпечується максимальна інтенсивність молоковиведення та мінімізується механічне навантаження на тканини дійки. Отримані результати можуть бути використані для вдосконалення конструкцій доїльних апаратів і підвищення ефективності машинного доїння.

Запропонована теоретична модель дозволяє комплексно оцінити взаємозв'язок між параметрами вакуумного режиму, характеристиками пульсації та механічним впливом дійкової гуми на тканини дійки. Результати дослідження можуть бути використані для обґрунтування енергоефективних і фізіологічно безпечних режимів роботи доїльних апаратів у сучасних системах машинного доїння.

Ключові слова: *машинне доїння, молоковіддача, доїльний апарат, параметри пульсації, дійкова гума, робочий вакуум, математичне моделювання, гідродинаміка, пружно-деформаційний стан, сфінктер дійки.*

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