



HYDRODYNAMIC REGULARITIES OF THE MECHANICAL MIXING PROCESS OF VISCOUS LIQUID MASSES

Andriy MARTYNYUK, Candidate of Technical Sciences, Associate Professor
Myroslav STECHYSHYN, Doctor of Technical Sciences, Professor
Mykola BORYS, Candidate of Technical Sciences, Associate Professor
Victor FEDORIV, Candidate of Technical Sciences, Associate Professor
Khmelnitskyi National University.

МАРТИНЮК Андрій Віталійович, к.т.н., доцент
СТЕЧИШИН Мирослав Степанович, д.т.н., професор
БОРИС Микола Михайлович, к.т.н., доцент
ФЕДОРІВ Віктор Михайлович, к.т.н., доцент
Хмельницький національний університет

The article investigates the influence of kinematic parameters on the velocity of grain material movement along the working surface of the rotor in a vibro-centrifugal separator. The relevance of the study is determined by the need to increase the efficiency of cleaning and sorting processes of grain mixtures through the use of centrifugal and vibration fields, which enhance the differences between particles in terms of specific mass and aerodynamic properties. The paper examines the mechanics of motion of an individual grain particle on the surface of a conical rotor performing simultaneous rotational and vibrational motion. The main forces acting on the particle during its movement are determined, including the centrifugal inertial force, the inertial force caused by vibrational motion, the gravitational force, the reaction force of the rotor surface, and the friction force between the particle and the working surface.

Based on the analysis of the conditions for continuous contact motion of a particle along the rotor surface, a differential equation describing its motion was obtained, and relationships for determining the displacement and velocity of grain particles depending on the vibration phase were derived. The conditions ensuring stable particle motion in the direction of expansion of the conical rotor with periodic stops were established. An analytical expression for determining the average velocity of grain material movement was obtained, taking into account the vibration amplitude and frequency, the angular velocity of rotor rotation, the cone angle of the working body, and the coefficient of friction between the material and the rotor surface.

The theoretical studies were supplemented by experimental investigations carried out on laboratory and pilot experimental installations, where the influence of rotor rotational speed, grain feed rate, and air flow parameters on the separation efficiency was analyzed. It was established that an increase in the vibration amplitude and frequency, as well as in the angular velocity of the rotor, leads to an increase in the average velocity of grain particle movement along the working surface. The obtained results can be used to substantiate the design and operating parameters of vibro-centrifugal separators and to develop high-performance grain cleaning machines.

Keywords: grain material, vibro-centrifugal separator, kinematic parameters, particle motion, vibration, centrifugal force, grain separation, grain cleaning.

Eq. 11. Fig. 2. Ref. 10.

1. Problem formulation

Modern food industry enterprises require efficient technological equipment capable of ensuring stable product quality while reducing energy consumption during production processes. One of the key operations in the production of curd masses is the mechanical mixing of viscous and viscoplastic liquid media in reactor-type apparatuses. Curd masses are produced using a mixing-based technological process and consist of mashed full-fat or semi-fat fresh cheese, sugar or powdered refined sugar, softened butter, liquid fractions, and various additives. The characteristics and quality parameters of the final product depend on the proportions of the initial ingredients, as well as on the temperature conditions and duration of the technological process.





Therefore, the efficiency of mechanical mixing significantly affects the homogeneity, rheological properties, and overall quality characteristics of the resulting product.

At the same time, the mixing of non-Newtonian media is accompanied by complex hydrodynamic processes, variations in viscosity during processing, and uneven distribution of flow velocities within the reactor volume. These factors lead to increased power consumption and reduced efficiency of conventional mixing systems. In addition, the mechanisms of hydrodynamic interaction and energy transfer during the production of curd masses remain insufficiently studied. Therefore, the development of approaches for modeling the mechanical mixing process and determining rational operating parameters of mixing equipment is an important scientific and practical task aimed at improving energy efficiency and intensifying technological processes in the food industry

2. Analysis of recent research and publications

A significant number of works by domestic and foreign scientists have been devoted to the study of the processes of hydrodynamics, heat and mass transfer during mixing of viscous and viscoplastic media in the food industry. The fundamental foundations of the rheology of food masses and the theory of mechanical mixing are laid down in classical works (Machikhin Yu.A., Gorbatyuk V.I., Steffe J., etc.).

Modern research (in particular, the works of Burdo O.G., Cherevko O.I., Gavrish A.V., as well as foreign authors such as Cullen P.J., Ein-Mozaffari F.) is mainly focused on optimizing the geometry of low-speed mixing devices (ribbon, screw, anchor, planetary mixers) used to work with non-Newtonian fluids. It has been proven that during the processing of such media, the effective viscosity depends exponentially on the shear rate, which leads to the formation of local zones of intense shear near the mixer blades and the emergence of large stagnant (dead) zones on the periphery of the apparatus [1-4].

3. The purpose of the article

The purpose of the study is to develop a generalized concept for modeling the process of mechanical mixing of viscous liquid media to increase equipment efficiency, reduce specific energy consumption, and ensure consistently high product quality.

4. Results and discussion

The proposed approach is based on a comprehensive consideration of the laws of fluid hydrodynamics, energy characteristics of mixing equipment and continuous changes in the rheological properties of the medium during the processing process.

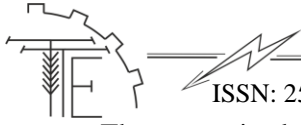
The primary apparatus used in curd mass production is the reactor - an agitated vessel in which the mixing of liquid media is carried out. The reactor accounts for the main share of energy consumption associated with the mixing process. During production, the reactor performs mixing at various operational stages, where the composition and temperature of the liquid medium change, leading to temporal variations in the density and viscosity of the medium, which in turn affect the power consumed by the mixing device [2]. Therefore, reducing the power consumption of the reactor remains an important and relevant task.

The production of curd masses involves the following main stages: loading the components according to the formulation proportions, heating to pasteurization temperature, pasteurization, cooling to the inoculation temperature, inoculation, and subsequent cooling [3, 4].

The technological process is carried out in a stainless steel reactor equipped with a jacket for heating and cooling, as well as an impeller for mixing the batch. The reactor is additionally provided with a cooling coil. Through the loading hatch, the calculated amount of formulation components, not exceeding 100 kg, is added to the reactor. With the impeller operating and at a temperature of 20 - 30 °C, the filler is introduced through a funnel. The components are dissolved at 20 - 30°C over 20 - 30 minutes under continuous mixing [5].

To assemble the mixture, liquid products (milk, cream, water) are added first, followed by concentrated dairy products, and finally dry ingredients (dry milk products, dry egg powder, sugar, stabilizers). It is recommended that dry ingredients be pre-mixed with sugar and a small amount of milk or water. The raw materials for the mixture should be combined at 35–40 °C to ensure the most complete and rapid dissolution of all components. The mixture is then filtered to remove any undissolved particles.

The mixture undergoes pasteurization, secondary filtration, homogenization, and cooling to the lowest possible positive temperatures. Pasteurization is performed at 85 °C with a holding time of 50–60 seconds, or at 92–95 °C without holding.



The pasteurized mixture is cooled to 2–4 °C using the cooling coil to initiate the maturation process, which lasts 4–48 hours. During storage, the mixture matures: the liquid fat solidifies, and the water-holding capacity of proteins and stabilizers increases. Maturation is an essential operation, and its duration depends on the hydrophilic properties of the stabilizer. When gelatin is added, the maturation process lasts at least 4 hours.

Upon completion, the curd mass is cooled to 25–40 °C by circulating chilled water through the reactor jacket. Finally, the curd mass is discharged into a drainage tank [6].

From the description of curd mass production, it follows that during all stages of the product preparation in the reactor, energy is consumed for mixing the liquid medium. Specifically, the total useful power, N_{gen} is expended, which represents the sum of the power contributions of individual operations during mixing:

$$N_{gen} = \sum_{i=1}^n \tau_i N_i, \quad (1)$$

where τ_i is the duration of an individual operation, N_i is the useful power consumed for mixing during that specific operation, n is the total number of operations, and i is the index of the individual operation ($i=1, 2, 3, \dots, n$).

The useful power of an individual operation N_i is determined by the following equation:

$$N_i = K_{N_i} \rho_c n_i^3 D_M^5, \quad (2)$$

where K_{N_i} is the power number representing the power consumed during the technological operation, ρ_c is the density of the medium during the given operation, n_i is the impeller rotational speed, and D_M is the impeller diameter.

The value of N_i is necessary to obtain results that allow for informed decision-making in reactor design. To achieve this, it is proposed to perform physical modeling of the mixing process under laboratory conditions, taking into account the characteristics of the actual reactor, followed by the transfer of technological properties from the model to the real system based on the theory of similarity.

For modeling the mechanical mixing of liquid curd masses, an aqueous starch suspension with a mass fraction of 6% was used. This allows the rheological properties of the mixture to be reproduced, approximating those of real curd systems, including their pseudoplastic behavior and the dependence of viscosity on shear rate [7].

To determine the density of the liquid medium ρ_c it is sufficient to use a model medium at a fixed temperature, namely the aqueous suspension, the density of which can be calculated using the following equation:

$$\frac{1}{\rho_c} = \frac{x}{\rho_{tv}} + \frac{1-x}{\rho_p} \quad (3)$$

where ρ_{tv} is the density of the solid starch particles, ρ_p is the density of water, and x is the mass fraction of starch in the suspension.

Regarding the type and diameter of the impeller, the same type of impeller was selected, while the diameter of the laboratory-scale impeller d_M was reduced by a factor k relative to the industrial impeller diameter D_M as follows:

$$k = D_M/d_M. \quad (4)$$

It is essential to maintain constant the following parameters: the ratio of the vessel diameter to the impeller diameter, the ratio of the impeller blade width to the vessel diameter, as well as the number of blades, the blade pitch angle relative to the horizontal, and the number and width of installed baffles.

The impeller rotational speed n_i remains unchanged.

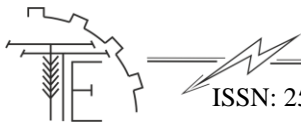
The power number K_{N_i} , consumed during a given operation, for the subsequent use of the experimental mixing data, is recommended to be calculated using the following formula [8]:

$$K_{N_i} = C Re_{M_i}^m, \quad (5)$$

where C and m are constant coefficients. Taking the logarithm of this relationship provides a linear form that is convenient for practical calculations [8]:

$$\lg K_{N_i} = \lg C + m \lg Re_{M_i}^m, \quad (6)$$

where $Re_{M_i}^m$ is the known value of the modified Reynolds number for the given operation.



To verify the aforementioned assumption and to enable its application in physical modeling of mixing processes during curd mass production, detailed investigations of the mixing process were conducted using a laboratory-scale experimental setup.

The experimental setup (Fig. 1) included an impeller system consisting of a tank (1), in which a vertically mounted motor-reducer (2) was installed, and a vessel (3) equipped with a mechanical frame impeller (4). The axis of the electric motor (2) was coaxial with the axis of the mechanical impeller (4).

A frequency converter connected to the motor allowed smooth adjustment of the impeller rotational speed. The rotational speed was measured using an electronic sensor (6). The power consumed for mechanical mixing was determined based on voltmeter and ammeter readings. The temperature of the liquid was recorded using a thermometer (5).

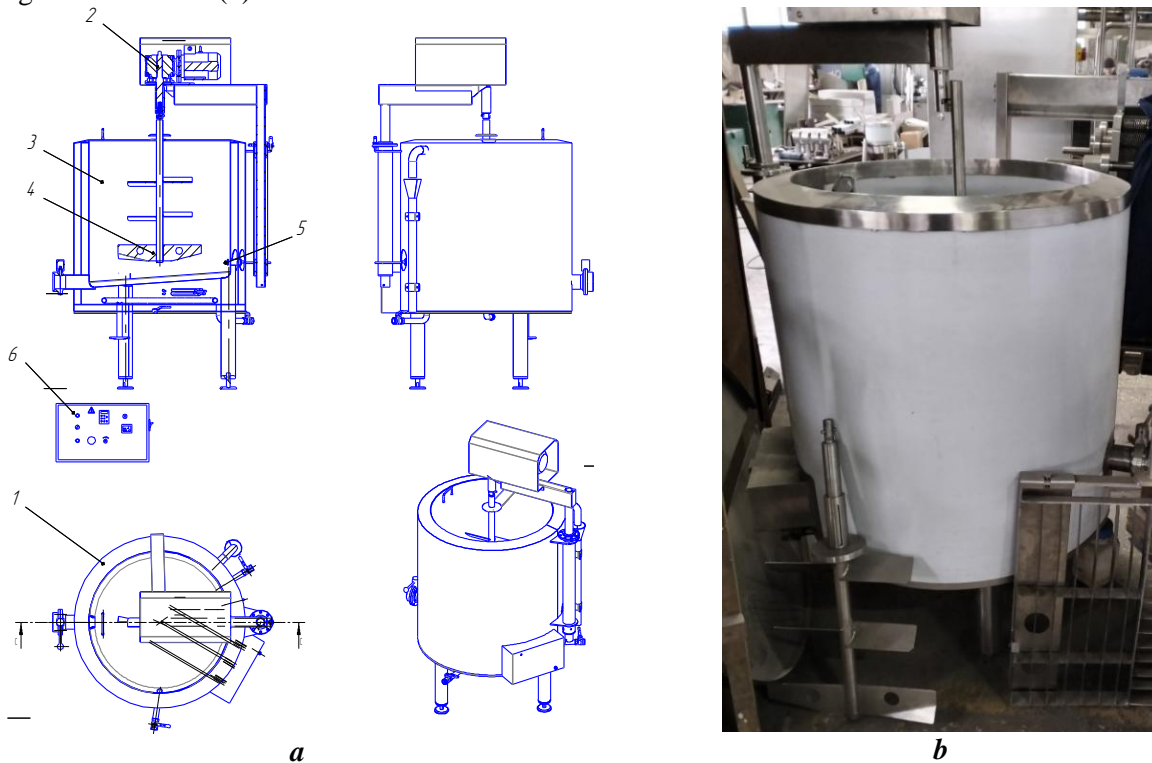


Fig. 1. Schematic of the laboratory experimental setup (a – schematic diagram, b – general view): 1 – tank; 2 – motor-reducer; 3 – vessel; 4 – mechanical impeller; 5 – temperature sensor; 6 – control panel

At the initial stage of the study, it was necessary to prepare an aqueous suspension of the required concentration and pour it into the vessel. A starch–water suspension was selected as the model medium for investigating mechanical mixing. Starch with a purity of not less than 99% of $(C_6H_{10}O_5)_n$ was used; it was preliminarily ground using a pestle and mortar to an average particle size of 1.5–2.5 μm , as determined by microscopy. The polymer $(C_6H_{10}O_5)_n$ exhibits low solubility, does not form crystalline hydrates, and does not chemically interact with water.

The starch had a specific surface area of 2000 m^2/kg and a density of $\rho_t = 1500 \text{ kg/m}^3$. The aqueous suspension was prepared at a water temperature of 20 °C by adding starch in an amount of 6 wt.% ($x = 0.02$). As a result, the total volume of the suspension was adjusted to 5 L and subsequently poured into a mixing vessel with a diameter of $D = 160 \text{ mm}$, which established a liquid level of $H = 150 \text{ mm}$.

The calculated density of the aqueous starch suspension was $\rho_p = 1110.05 \text{ kg/m}^3$.

The dynamic viscosity coefficient μ_s of the aqueous suspension, which depends on the concentration of solid particles, was determined using the following equation:

$$\mu_c = \mu_p(1 + 2,5\varphi), \quad (7)$$

where φ – the volumetric fraction of solid particles in the suspension, calculated using the following equation:

$$\varphi = x/x + \frac{\rho_t}{\rho_p}(1 - x) = 0,102. \quad (8)$$

The dynamic viscosity coefficient of the aqueous suspension was finally determined to be



$\mu_s = 1.250 \times 10^{-6} \text{ Pa}\cdot\text{s}$. A frame-type impeller was investigated, with a diameter of $d_m = 400 \text{ mm}$. By design, the frame impeller consisted of two mutually perpendicular blades arranged horizontally, mounted in three tiers along the height, with each blade having a width of 60 mm. The vessel was equipped with baffles (Fig. 1).

The experimental procedure was as follows. Prior to the experiment, the handle of the autotransformer was set to zero. The experimental setup was then connected to the power supply. By gradually rotating the autotransformer handle, the voltage was set to 50 V, as indicated by the voltmeter. After reaching a steady operating regime of the impeller, the readings of the voltmeter (U, V), ammeter (I, A), and electronic tachometer (n, rpm) were recorded. The same procedure was repeated for voltmeter readings of 60 V, 70 V, 80 V and 90 V. The experiment was conducted three times, and the instrument readings were documented. Upon completion of the experiment, the autotransformer handle was smoothly returned to zero, and the setup was disconnected from the power supply. The temperature of the aqueous suspension, t , was measured before and after the experiment with the impeller turned off.

It should be noted that at rotational speeds exceeding 130 rpm, a vortex formed in the mixing vessel, indicating a probable decrease in mixing intensity and efficiency.

Knowing the impeller diameter d_m , the modified Reynolds number was calculated. Subsequently, based on the experimental values of U and I , the power consumed by the impeller for mechanical mixing was determined:

$$N = UI \cos \alpha \eta_{dv}, \quad (9)$$

where U is the voltage (V); I is the current (A); $\cos \alpha$ is the power factor; and η is the efficiency of the electric motor. In the calculations, $\eta = 0.8$ and $\cos \alpha = 0.85$ were assumed.

The experimental power number was determined using the following equation:

$$K_{Ni} = N / \rho n^3 d_M^5, \quad (10)$$

The principal parameter to be considered in mixing processes is the power consumed by the mixing device, which depends on numerous factors. Therefore, the objective of this experimental study was to investigate the mixing process, specifically to determine the influence of geometric and technological parameters on the power consumption of the impeller, as well as to establish a mathematical relationship between the power number and the modified Reynolds number.

Based on the experimental results, a graphical dependence of $\lg Re_m$ versus $\lg K_n$ was obtained using Microsoft Excel, which appeared as a straight line with a coefficient of determination $R^2 = 0.9918$. The resulting relationship can be expressed as follows:

$$K_{Ni} = 4,7311 \cdot 10^{16} \cdot Re_M^{-1,82113} \quad (11)$$

The graphical relationship between the logarithm of the impeller power number, $\lg K_n$, and the logarithm of the modified Reynolds number, $\lg Re_m$, is presented in Fig. 2.

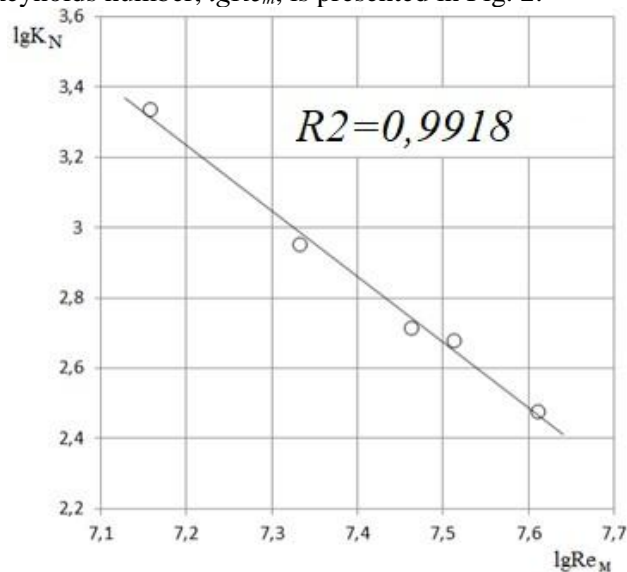
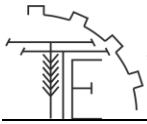


Fig. 2. Graph of the dependence of the logarithm of the impeller power number, $\lg K_n$, on the logarithm of the modified Reynolds number, $\lg Re_m$



5. Conclusion

Based on the results of the conducted studies on physical modeling of the mechanical mixing process of liquid media in a reactor for the production of cheese masses, the following conclusions can be drawn:

The total useful power consumed by the reactor during the mixing of cheese masses is determined by the cumulative number of technological operations, the duration of each individual operation, and the magnitude of useful power expended at each stage of the process.

A generalized concept for modeling the process of mechanical mixing of liquid media in a reactor for the production of cheese masses has been proposed.

A methodology for processing experimental data obtained from a laboratory experimental setup simulating the mechanical mixing process has been developed.

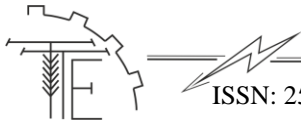
An empirical relationship for calculating the mixing power number as a function of the modified Reynolds number has been established, which makes it possible, within the scope of the conducted experiments, to predict power consumption for the mixing process

References

1. Rayner, M., & Dejmek, P. (2015). *Engineering aspects of food emulsification and homogenization*. CRC Press, Taylor & Francis Group. [in English].
2. Barros, P. L., Ein-Mozaffari, F., & Lohi, A. (2022). Gas dispersion in non-Newtonian fluids with mechanically agitated systems: A review. *Processes*, 10(2), 275. <https://doi.org/10.3390/pr10020275> [in English].
3. Wang, S., Zhang, Y., Liu, X., & Li, J. (2022). Simulation analysis of power consumption and mixing performance of propeller mixers in pseudoplastic non-Newtonian fluids. *Energies*, 15(13), 4561. <https://doi.org/10.3390/en15134561> [in English].
4. Raut, A. S., & Chavan, V. M. (2021). Computational fluid dynamics used by mixing vessels for predicting hydrodynamic behaviour of mixture: An overview. *Materials Today: Proceedings*, 47, 5031–5038. <https://doi.org/10.1016/j.matpr.2021.05.425> [in English].
5. Lins Barros, P., Ein-Mozaffari, F., & Lohi, A. (2022). Empirical correlations and hydrodynamics in mixing of non-Newtonian fluids. *Processes*, 10(5), 913. <https://doi.org/10.3390/pr10050913> [in English].
6. Tamburini, A., Cipollina, A., Micale, G., & Brucato, A. (2022). Liquid mixing time and gas distribution in aerated multiple impeller stirred tanks. *Chemical Engineering Research and Design*, 184, 501–512. <https://doi.org/10.1016/j.cherd.2022.06.020> [in English].
7. Cullen, P. J. (Ed.). (2009). *Food mixing: Principles and applications*. Wiley-Blackwell. <https://doi.org/10.1002/9781444318210> [in English].
8. Paul, E. L., Atiemo-Obeng, V. A., & Kresta, S. M. (Eds.). (2004). *Handbook of industrial mixing: Science and practice*. Wiley. <https://doi.org/10.1002/0471451452> [in English].
9. Kresta, S. M., Etchells, A. W., Dickey, D. S., & Atiemo-Obeng, V. A. (2016). *Advances in industrial mixing: A companion to the Handbook of industrial mixing*. Wiley. <https://doi.org/10.1002/9781118885239> [in English].
10. Mykhalevych, A., Turchyn, I., & Kolesnyk, A. (2022). Functional and technological properties of protein ingredients in whey ice cream. *Ukrainian Journal of Food Science*, 10(1), 45–54. <https://doi.org/10.24263/2310-1008-2022-10-1-6> [in English].

ГІДРОДИНАМІЧНІ ЗАКОНОМІРНОСТІ ПРОЦЕСУ МЕХАНІЧНОГО ПЕРЕМІШУВАННЯ В'ЯЗКИХ РІДКИХ МАС

Широке використання сирних мас як складової у різних напрямках молочної промисловості зумовлене відносною простотою технології їх отримання, доступністю сировини та можливістю гнучкого регулювання структурно-механічних властивостей готового продукту. Ключовим елементом технологічної лінії є реактор – апарат з мішалкою, в якому реалізується процес інтенсивного механічного перемішування в рідкому або в'язкопластичному середовищі. Саме цей вузол визначає ефективність гідродинамічних процесів, рівномірність розподілу багатокomпонентних сумішей, швидкість тепло- та масообміну, а також формування кінцевої структури сирної маси. Під час механічної обробки сирні маси проявляють яскраво виражені неньютонівські (псевдопластичні) властивості, зокрема аномалію в'язкості та наявність граничної напруги зсуву. Це суттєво ускладнює прогнозування полів швидкостей у робочому об'ємі апарата, оскільки ефективна в'язкість



середовища безперервно змінюється залежно від градієнта швидкості зсуву та тривалості перемішування.

Встановлено, що на роботу реактора припадає основна частка енергоспоживання під час виробництва, що обумовлює критичну необхідність оптимізації режимів його роботи. Традиційні емпіричні методи розрахунку часто призводять до невиправданого завищення встановленої потужності приводів. Загальна корисна потужність, яка витрачається при перемішуванні, залежить від сукупності технологічних операцій, тривалості їх виконання, а також локальних енергетичних витрат на кожному етапі. При цьому визначальну роль відіграють мінливі реологічні властивості середовища, тип і просторова геометрія мішалки, частота її обертання та встановлений режим течії (ламінарний, перехідний або турбулентний).

У роботі представлено узагальнену концепцію моделювання процесу механічного перемішування рідких середовищ у молочній промисловості при виробництві сирних мас. Для розв'язання задачі використано апарат теорії подібності у поєднанні з рівняннями збереження маси та імпульсу. Запропонований підхід базується на комплексному врахуванні гідродинамічних закономірностей руху рідини, модифікованих критеріїв потужності для неньютонівських систем, енергетичних характеристик обладнання та еволюції структурних властивостей середовища в процесі обробки.

Реалізація цієї концепції дозволяє на стадії проєктування або модернізації обґрунтовано вибирати раціональні конструктивно-кінематичні параметри апаратів. Це дає змогу суттєво знизити питомі енерговитрати, запобігти небажаному руйнуванню білкової матриці (синерезису) та забезпечити високу гомогенність і стабільну якість кінцевого продукту.

Ключові слова: сирні маси, реактор, мішалка, корисна потужність, перемішування, фізичне моделювання.

Ф. 11. Рис. 2. Літ. 10.

INFORMATION ABOUT THE AUTHORS

Andriy MARTYNYUK – Candidate of Technical Sciences, Associate Professor of the Department of Branch Mechanical Engineering and Agricultural Engineering of the Khmelnytskyi National University (11, Instytutska St., Khmelnytskyi, 29016, Ukraine, e-mail: avmart@khnmu.edu.ua, <https://orcid.org/0000-0001-8277-1308>).

Myroslav STECHYSHYN – Doctor of Technical Sciences, Professor of the Department of Branch Mechanical Engineering and Agricultural Engineering of the Khmelnytskyi National University (11, Instytutska St., Khmelnytskyi, 29016, Ukraine, e-mail: miro011951@gmail.com, <https://orcid.org/0000-0001-5780-2790>).

Mykola BORYS – Candidate of Technical Sciences, Associate Professor of the Department of Branch Mechanical Engineering and Agricultural Engineering of the Khmelnytskyi National University (11, Instytutska St., Khmelnytskyi, 29016, Ukraine, e-mail: myk0borys@gmail.com, <https://orcid.org/0000-0003-0677-9505>).

Victor FEDORIV – Candidate of Technical Sciences, Associate Professor of the Department of Branch Mechanical Engineering and Agricultural Engineering of the Khmelnytskyi National University (11, Instytutska St., Khmelnytskyi, 29016, Ukraine, e-mail: fedoriv55@ukr.net, <https://orcid.org/0000-0002-4499-0910>).

МАРТИНЮК Андрій Віталійович – кандидат технічних наук, доцент кафедри галузевого машинобудування та агроінженерії Хмельницького національного університету (вул. Інститутська, 11, м. Хмельницький, 29016, Україна, e-mail: avmart@khnmu.edu.ua, <https://orcid.org/0000-0001-8277-1308>).

СТЕЧИШИН Мирослав Степанович – доктор технічних наук, професор кафедри галузевого машинобудування та агроінженерії Хмельницького національного університету (вул. Інститутська, 11, м. Хмельницький, 29016, Україна, e-mail: miro011951@gmail.com, <https://orcid.org/0000-0001-5780-2790>).

БОРИС Микола Михайлович – кандидат технічних наук, доцент кафедри галузевого машинобудування та агроінженерії Хмельницького національного університету (вул. Інститутська, 11, м. Хмельницький, 29016, Україна, e-mail: myk0borys@gmail.com, <https://orcid.org/0000-0003-0677-9505>).

ФЕДОРІВ Віктор Михайлович – кандидат технічних наук, доцент кафедри галузевого машинобудування та агроінженерії Хмельницького національного університету (вул. Інститутська, 11, м. Хмельницький, 29016, Україна, e-mail: fedoriv55@ukr.net, <https://orcid.org/0000-0002-4499-0910>).