

JUSTIFICATION OF THE OPTIMAL SPEED REGIME FOR A TRACTOR-IMPLEMENT UNIT BASED ON TRACTION-ENERGY AND TECHNOLOGICAL PERFORMANCE INDICATORS

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The article provides a theoretical substantiation of the rational operating speed of a machine-tractor unit based on a comprehensive consideration of traction, energy, and technological performance indicators. The influence of operating speed on draft resistance, required drawbar power, field capacity, wheel slip, traction efficiency, hourly and specific fuel consumption, and specific energy consumption is determined. A generalized mathematical model of the draft resistance of an agricultural machine is proposed, in which the total resistance is divided into static and dynamic components while accounting for operating speed, working width, and tillage depth. The dynamic component of draft resistance is represented by a power-law dependence on operating speed, which makes it possible to describe the nonlinear increase in energy demand as the travel speed of the unit increases. Analytical relationships are obtained for determining drawbar power, theoretical and actual field capacity, wheel slip coefficient, traction efficiency, fuel consumption, and specific energy consumption. Numerical simulation demonstrates that an increase in operating speed improves field capacity only up to a certain limit, beyond which the increasing dynamic component of draft resistance, wheel slip, and power demand leads to deterioration in energy performance. The existence of a rational speed range is substantiated, within which a favorable balance between field capacity, fuel efficiency, specific energy consumption, and the required quality of the technological operation can be achieved. A criterion-based approach to selecting the rational operating speed is proposed. It is based on maximizing actual field capacity subject to specified constraints on specific fuel consumption, wheel slip, available drawbar power, and technological quality. The obtained theoretical relationships and simulation results can be used to substantiate operating modes of machine-tractor units, improve the efficiency of tractor–implement matching, and reduce energy consumption during mechanized agricultural operations.

Keywords: machine-tractor unit, operating speed, draft resistance, drawbar power, field capacity, specific fuel consumption, energy efficiency, wheel slip, specific energy consumption, rational operating mode.

Eq. 17. Fig. 7. Table. 2. Ref. 10.

1. Problem formulation

The efficiency of agricultural machinery usage is largely determined by the well-founded selection of operating modes for the tractor-implement unit. Among operational parameters, working speed plays a pivotal role, as it is directly linked to productivity, draft load, required engine power, fuel consumption, and the quality of the technological operation.

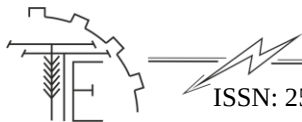
As the unit's travel speed increases, its theoretical productivity generally rises. At the same time, however, the draft resistance of the implement, drive-mechanism slippage, and the demand for draft power may also increase. Studies on tractor draft characteristics confirm that speed is a key parameter for evaluating the efficiency of power transmission from the engine to the working tool and the soil [1].

It is particularly important to determine not the maximum possible speed, but rather an optimal speed that ensures the required quality of the technological operation while minimizing energy expenditure. This approach aligns with the modern trend of optimizing agricultural machinery performance based on actual data regarding load, fuel consumption, and productivity [2; 3].

2. Analysis of recent research and publications

The impact of travel speed on the operational and energy characteristics of tractor-implement units has been investigated by numerous researchers. Md-Tahir et al. experimentally established the relationships between speed, draft force, wheel slip, drawbar power, fuel consumption, and productivity during tillage





operations [1]. Kim et al. demonstrated that the choice of gear and corresponding speed significantly affects the mechanical load on the engine and the tractor's fuel efficiency [2].

Pitla et al. proposed an approach for assessing actual tractor loads under field conditions using CAN-bus data [3]. Pentoš employed soft computing methods to analyze tractor traction characteristics under various soil conditions [4]. Studies modeling draft force during plowing have established that the draft load is determined by the interaction of tillage depth, soil properties, and travel speed [5]. Kuvaev et al. examined the interrelationships among working width, draft resistance, speed, productivity, and specific fuel consumption [6].

Despite the substantial body of experimental research, the literature lacks generalized theoretical models capable of simultaneously analyzing the impact of speed on draft resistance, power, productivity, fuel consumption, wheel slip, and work quality. It is precisely this gap that underscores the relevance of the present study.

3. The purpose of the article

The aim of the study is to provide a theoretical basis for the patterns governing the influence of the machine-tractor unit's travel speed on key energy and operational performance indicators, and to establish a criterion-based approach for determining the optimal speed regime.

To achieve this aim, the following steps are required:

- develop a mathematical model of draft resistance, distinguishing between static and dynamic components;
- derive analytical relationships expressing draft power, productivity, and specific fuel consumption as functions of speed;
- investigate the impact of speed on wheel slip and traction efficiency;
- conduct numerical modeling and generate graphical and tabular representations of the relationships;
- formulate a criterion for selecting the optimal operating regime for the machine-tractor unit.

4. Results and discussion

The draft resistance of an agricultural machine is a function of the design of the working elements, the working width, the tillage depth, the physical-mechanical properties of the soil, and the travel speed. In general terms, it can be expressed as the sum of static (speed-independent) and dynamic (speed-dependent) components:

$$R_t(V, h, B) = R_0(h, B) + R_d(V, h, B), \quad (1)$$

where R_0 is the static component, kN; R_d is the dynamic component, kN; h is the tillage depth, m; B is the working width, m; and V is the operating speed, m/s (or km/h).

The static component for tillage implements is often approximated by a linear relationship with depth and width:

$$R_0 = (a_0 + a_1 \cdot h) \cdot B, \quad (2)$$

where a_0 and a_1 are empirical coefficients depending on the soil type and the design of the working elements.

The dynamic component is caused by inertial forces and the increase in resistance associated with the more rapid breakdown of soil aggregates. As a first approximation, it can be described by a quadratic relationship with respect to speed:

$$R_d = k \cdot B \cdot V^n, \quad (3)$$

where k is the dynamic resistance coefficient; n is the exponent (typically $n \approx 1.5$ – 2.5). For generalized calculations, $n = 2$ is often assumed:

$$R_t = R_0 + k \cdot V^2. \quad (4)$$

Figure 1 clearly illustrates the structure of tractive resistance, showing that the share of the dynamic component increases significantly as speed rises.

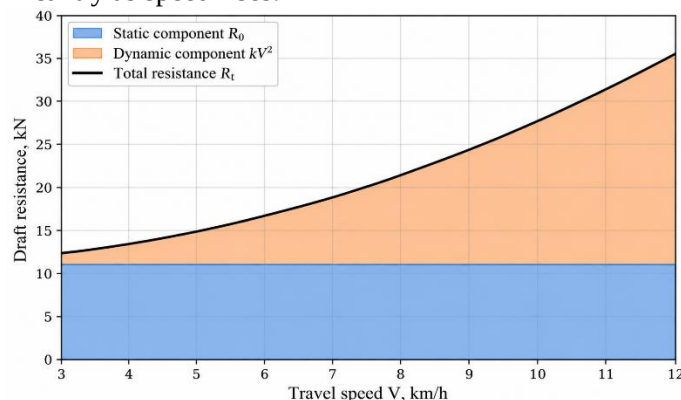


Fig. 1. Structure of tractive resistance: static (R_0) and dynamic (kV^2) components.



The required tractive power is determined by the product of tractive resistance and speed:

$$N_t = (R_t \cdot V) / 3,6 = (R_0 \cdot V + k \cdot V^3) / 3,6, \quad (5)$$

where N_t is the tractive power (kW) and V is the speed (km/h). The expression shows that power depends on speed raised to both the first and third powers. Therefore, even a moderate increase in speed—given a significant dynamic component—leads to a sharp rise in power demand.

Under conditions of constant resistance ($k = 0$), doubling the speed requires approximately doubling the tractive power. If $k > 0$, however, the actual demand increases even more sharply [8].

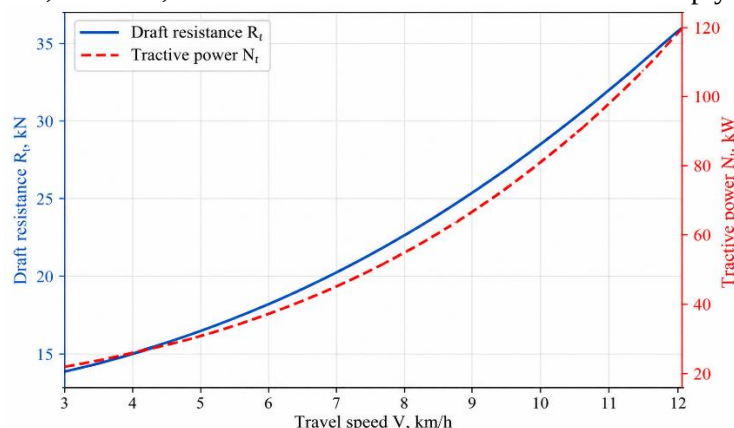


Fig. 2. Dependence of draft resistance and draft power on travel speed

For a comprehensive analysis of the impact of tillage speed and depth, we will use the following model:

$$R_t(V, h) = (a_0 + a_1 \cdot h) + k \cdot V^2. \quad (6)$$

The spatial surface constructed according to this dependence (Fig. 3) shows that the largest values of traction resistance correspond to a simultaneous increase in speed and depth. This confirms the need for comprehensive consideration of both parameters when choosing the operating mode.

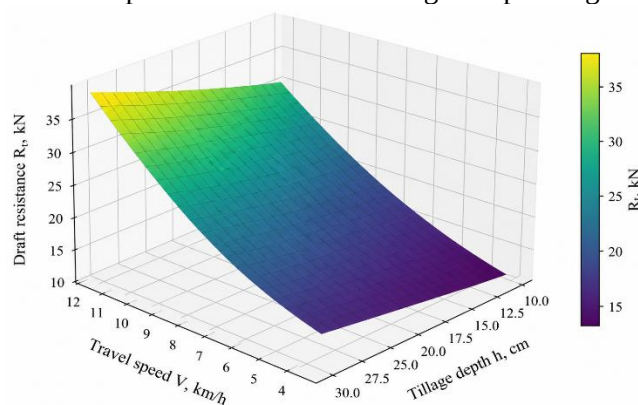


Fig. 3. Spatial dependence of draft resistance on speed and tillage depth

The theoretical productivity of the machine-tractor unit is determined as:

$$W_t = 0,1 \cdot B \cdot V \cdot \tau, \quad (7)$$

where W_t is the theoretical productivity, ha/h; B is the working width, m; V is the speed, km/h; and τ is the shift time utilization coefficient (0.7–0.85).

Actual productivity accounts for losses due to wheel slip:

$$W\phi = W_t \cdot (1 - \delta) = 0,1 \cdot B \cdot V \cdot \tau \cdot (1 - \delta), \quad (8)$$

where δ is the slip coefficient (expressed as a fraction of unity).

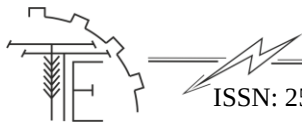
The slip coefficient is determined by the formula:

$$\delta = (V_0 - V) / V_0, \quad (9)$$

where V_0 is the theoretical speed without wheel slip. To approximate the relationship $\delta(V)$ in the calculations, an increasing power function was used, reflecting the increase in wheel slip as the tractive load rises:

$$\delta(V) = \delta_0 + \alpha \cdot (V - V_{min})^m, \quad (10)$$

The traction efficiency coefficient can be estimated as:



$$\eta_t \approx \eta_0 \cdot (1 - \beta \cdot \delta), \quad (11)$$

where η_0 is the base efficiency of the transmission and propulsion units, and β is the slip influence coefficient.

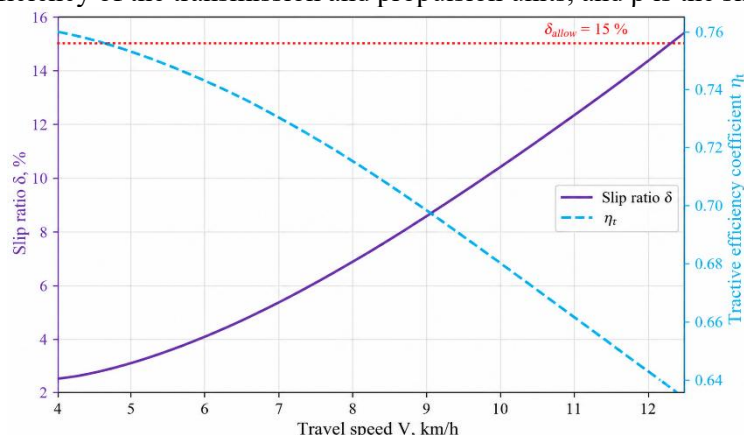


Fig. 4. Effect of speed on the slip coefficient and tractive efficiency

Hourly fuel consumption can be approximated by a linear relationship with effective power:

$$G = G_0 + c \cdot N_e \approx G_0 + c \cdot N_t / \eta_t, \quad (12)$$

where G_0 is the idling fuel consumption, kg/h; c is the specific fuel consumption, kg/(kW·h).

Specific fuel consumption per unit area:

$$g_{pl} = G / W\phi. \quad (13)$$

Specific energy consumption (kWh/ha) is defined as:

$$E = N_t / W\phi. \quad (14)$$

The dependencies of productivity, specific fuel consumption, and specific energy consumption on speed are shown in Figs. 5 and 6. A zone where specific indicators reach a minimum is evident; this zone corresponds to the rational speed range.

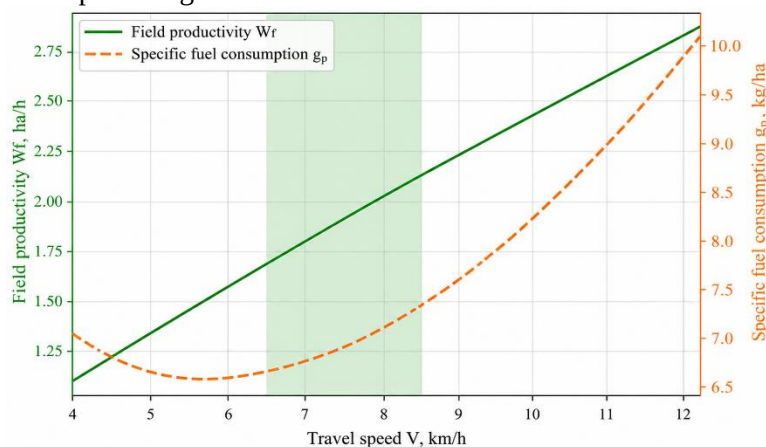


Fig. 5. Dependence of actual productivity and specific fuel consumption on speed

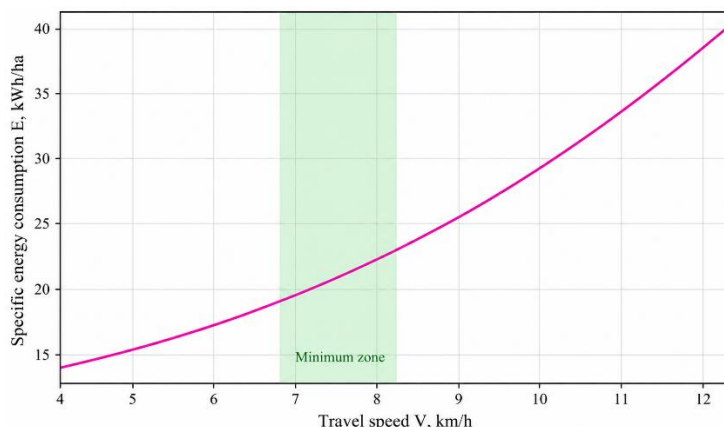


Fig. 6. Dependence of specific energy consumption on the unit's travel speed



The diagram of forces acting on the mobile power unit during motion is shown in Fig. 7. The equilibrium condition for tractive forces is:

$$F_t \geq R_t + F_p, \quad (15)$$

where F_t is the tractor's realized tractive effort; F_p is the tractor's rolling resistance.

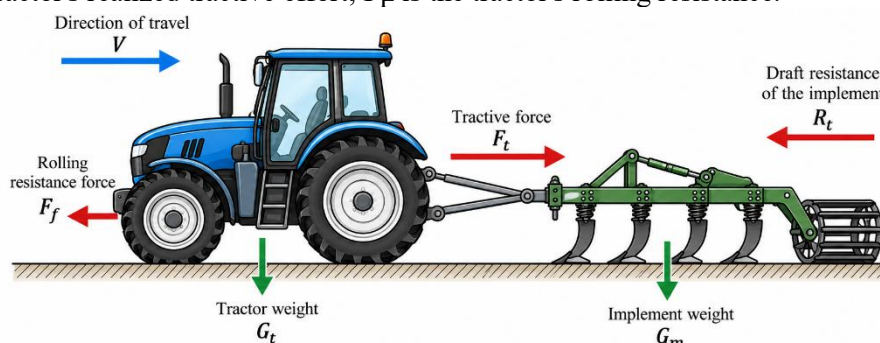


Fig. 7. Diagram of a machine-tractor unit and the main forces acting during movement

To illustrate the proposed relationships, numerical modeling was performed using the following initial data: working width $B = 3.6$ m; time utilization factor $\tau = 0.78$; static resistance component $R_0 = 11.5$ kN; dynamic resistance coefficient $k = 0.16$; and slip parameters selected such that at $V = 10$ km/h, $\delta \approx 12-14\%$. The calculation results are summarized in Table 1.

Table 1.

Calculated values of key MTU performance indicators versus speed

V, км/год	R _t , кН	N _t , кВт	W _ф , га/год	δ, %	G, кг/год	g _{pl} , кг/га	E, кВт·год/га
4,0	14,1	15,6	1,08	3,0	7,6	7,1	14,5
5,0	15,5	21,5	1,33	4,2	8,9	6,7	16,2
6,0	17,3	28,8	1,57	5,6	10,5	6,7	18,4
7,0	19,3	37,6	1,79	7,2	12,5	7,0	21,0
8,0	21,7	48,3	1,99	9,0	14,8	7,4	24,3
9,0	24,5	61,1	2,16	11,1	17,6	8,2	28,3
10,0	27,5	76,4	2,30	13,4	21,0	9,1	33,2
11,0	30,9	94,3	2,41	16,0	24,9	10,3	39,1
12,0	34,5	115,1	2,49	18,8	29,5	11,9	46,2

Table 1 shows that minimum specific fuel consumption values (approximately 6.7 kg/ha) occur in the speed range of 5–6 km/h. However, productivity is relatively low in this range. In the 7–8 km/h range, an acceptable level of specific fuel consumption (7.0–7.4 kg/ha) is maintained, accompanied by significantly higher productivity and wheel slip that remains within permissible limits ($\delta < 10\%$). Any further increase in speed leads to a rapid rise in both specific fuel consumption and wheel slip.

Table 2 presents the effect of tillage depth on draft resistance at a constant speed of $V = 8$ km/h.

Table 2.

Effect of tillage depth on draft resistance ($V = 8$ km/h)

Глибина h, см	R ₀ , кН	R _д , кН	R _t , кН
12	9,4	10,2	19,6
16	11,2	10,2	21,4
20	13,0	10,2	23,2
24	14,8	10,2	25,0
28	16,6	10,2	26,8

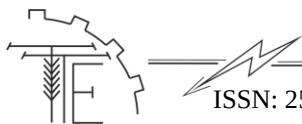
Based on the analysis performed, the problem of selecting the optimal speed can be formulated as a constrained optimization problem:

$$V_{onm} = \arg \max W\phi(V), \quad (16)$$

under the conditions of:

$$g_{pl}(V) \leq g_{don}, \quad \delta(V) \leq \delta_{don}, \quad Q(V) \geq Q_{don}, \quad N_t(V) \leq N_{m.don}, \quad (17)$$

where g_{perm} is the permissible specific fuel consumption; δ_{perm} is the permissible wheel slip



(typically 12–15%); Q is the generalized quality indicator of the technological operation; and $N_{t,perm}$ is the permissible tractor drawbar power.

The quality of the technological operation $Q(V)$ for various machines may encompass depth uniformity, the degree of soil pulverization, seed distribution uniformity, and so forth. Excessive speed generally leads to a decline in quality; therefore, the constraint $Q(V) \geq Q_{perm}$ is mandatory.

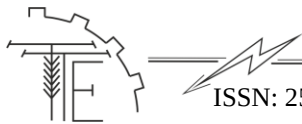
Based on the simulation results for the example under consideration, the optimal speed range is approximately 6.5–8.5 km/h. This range ensures a balance of high productivity, acceptable specific fuel consumption, and permissible wheel slip.

5. Conclusion

1. A generalized mathematical model of the tractive resistance of a mobile machinery unit (MMU) has been developed, distinguishing between static and dynamic components; this enables the analysis of speed's influence on energy performance indicators.
2. Analytical relationships have been derived expressing tractive power, actual productivity, specific fuel consumption, the wheel slip ratio, and specific energy consumption as functions of travel speed.
3. Numerical modeling revealed the existence of an optimal speed range (6.5–8.5 km/h in the case study) that achieves a balance between productivity, energy consumption, and wheel slip.
4. A criterion-based approach to selecting the optimal speed has been proposed, formulated as a constrained optimization problem involving limits on specific fuel consumption, wheel slip, work quality, and tractor power.
5. Future research should focus on the experimental identification of model coefficients for specific types of machinery and soil conditions, as well as on integrating the proposed approach into MMU onboard monitoring systems.

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ОБҐРУНТУВАННЯ РАЦІОНАЛЬНОГО ШВИДКІСНОГО РЕЖИМУ МАШИННО-ТРАКТОРНОГО АГРЕГАТУ ЗА ТЯГОВО-ЕНЕРГЕТИЧНИМИ ТА ТЕХНОЛОГІЧНИМИ ПОКАЗНИКАМИ

У статті теоретично обґрунтовано раціональний швидкісний режим машинно-тракторного агрегату з урахуванням комплексу тягово-енергетичних і технологічних показників. Встановлено характер впливу робочої швидкості на тяговий опір, необхідну тягову потужність, продуктивність, буксування рушіїв, тягову ефективність, годинну та питому витрату палива і питомі енерговитрати. Запропоновано узагальнену математичну модель тягового опору сільськогосподарської машини, яка передбачає його поділ на статичну та динамічну складові та враховує вплив швидкості руху, ширини захвату і глибини обробітку. Динамічну складову тягового опору представлено степеневою залежністю від швидкості, що дає змогу відобразити нелінійне зростання енергетичного навантаження зі збільшенням швидкості руху агрегату. Отримано аналітичні залежності для визначення тягової потужності, теоретичної та фактичної продуктивності, коефіцієнта буксування, тягової ефективності, витрати палива та питомих енерговитрат. За результатами чисельного моделювання встановлено, що підвищення швидкості забезпечує зростання продуктивності лише до певної межі, після якої збільшення динамічної складової опору, буксування та потреби в потужності призводить до погіршення енергетичних показників. Обґрунтовано наявність раціонального діапазону швидкостей, у межах якого забезпечується сприятливе співвідношення між продуктивністю, паливною економічністю, енерговитратами та якістю виконання технологічної операції. Запропоновано критеріальний підхід до вибору швидкісного режиму, що передбачає максимізацію фактичної продуктивності за встановлених обмежень щодо питомої витрати палива, буксування, тягової потужності та технологічної якості. Отримані результати можуть бути використані для обґрунтування режимів експлуатації машинно-тракторних агрегатів та підвищення енергетичної ефективності механізованих технологічних операцій у сільськогосподарському виробництві.

Ключові слова: машинно-тракторний агрегат, робоча швидкість, тяговий опір, тягова потужність, продуктивність, питома витрата палива, енергоефективність, буксування, питомі енерговитрати, раціональний режим.

Ф. 17. Рис. 7. Табл. 2. Літ. 10.

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